Abstract

GridPix is a gas filled detector fabricated directly on top of a pixel readout chip. Ionisation electrons from charged particles induce electron avalanches which are sensed by the bond pads of the chip. The current chip, Timepix3, has roughly circular pads that are 18 µm in diameter compared to 55 × 55 µm² pixels, so there is plenty room to enlarge them. In this research, we developed an extensive simulation of GridPix on Timepix3 and found that the readout channels only see 54(7) % of the total charge produced in an avalanche. To improve the charge collection efficiency, we modelled larger pads that can be fabricated on top of the existing ones. We found that a 50 µm diameter pad increases the charge collection efficiency to 96.7(7) %, without introducing significant crosstalk between pixels.
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A Analog front-end model

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Introduction

GridPix is a type of gaseous pixel detector that is made by fabricating a microscopic mesh—called a grid—on top of a pixel chip. Charged particles traversing the gas volume of this detector ionise molecules along their trajectories. After drifting to the grid, the ionisation electrons are amplified in electron avalanches by a factor called the gas gain. By measuring both the position and time of these avalanches, gaseous pixel detectors can measure the trajectories in three dimensions. Additionally, the number of ionisations contains information about the energy loss and could provide a mechanism for particle identification.

Operating a GridPix with low gas-gain has several advantages. It increases the durability of the detector by lowering the chance of discharges. It also makes the detector more tolerant to high rates of incoming particles by reducing charge buildup on the resistive protection layer which covers the pixel chip. However, a low gas-gain also leads to smaller signals, which decreases the detection efficiency. Furthermore, it decreases the accuracy of the time measurement because of so-called timewalk effects.

To mitigate the negative effects associated with a low gas-gain, it is important to optimise the charge collection efficiency to get the largest feasible signal from an electron avalanche. The shape and size of the sensing electrode, as well as other electrodes and materials surrounding it, determine how effective the electrode is at picking up an avalanche signal. In a gaseous pixel detector, the bump bond pads on a pixel chip function as sensing electrodes. However, unless a pixel chip is specifically designed for gas, the size of these pads is generally kept to a minimum—they only need to be large enough for bump bonding. The pixel chip currently used for GridPix is Timepix3. This is a general purpose chip and has relatively small pads: they are roughly circular and 18 µm in diameter compared to 55 × 55 µm² pixels. Fabricating larger pads on top of the pixel plane might increase the charge collection efficiency.

The main subject of this research is developing a simulation of GridPix on Timepix3 and using it to study pixel pad enlargement. We will investigate how much we can increase the charge collection efficiency by enlarging the pads, and how this affects the detection efficiency. In addition, we will look at possible unwanted consequences such as crosstalk or an increase in noise.

First, section 1 gives a brief description of gaseous detectors. It also discusses GridPix in the broader context of micro-pattern gaseous detectors. Next, section 2 describes in detail how we will model GridPix and Timepix3. We do this before covering more theoretical aspects required for our simulations. This way, the examples in those sections will be clearer and more meaningful.

In section 3 we will look at signal induction and define more precisely what I mean by the charge collection efficiency. In section 4, we continue by discussing how the analog front-end of Timepix3 responds to the induced signals. In section 5 we will look at how the resistive protection layer affects the signal induction.

Section 6 covers how we calculate electric fields using Gmsh and Elmer. In section 7 we go over the simulations. It describes how we produce avalanches in Garfield++ and how we use them to perform our measurements. We present the results in section 8 and discuss them in section 9.
1 Gaseous pixel detectors

When a charged particle traverses the gas volume of a gaseous detector it creates a limited number of electron-ion pairs along its track. An electric field transports the ionised electrons to one or more anodes. The signals that the drift of these electrons induces is generally too weak to detect because they are obscured by noise. For example, the preamplifier in Timepix3 has an equivalent noise charge (ENC) of roughly 75 $e^{-}$. In order to detect the initial electrons, they need to be amplified. This is achieved by a stronger electric field region where they gain enough energy to ionise the gas molecules. This process leads to an electron avalanche which induces a large enough signal to be measured. Further description of the workings of gaseous detectors can be found in [1].

1.1 Initial ionisation

The initial ionisation along a track can be divided into primary and secondary ionisation. In primary ionisation the particle traversing the detector directly ionises the gas molecules:

\[ p^{\pm} + G \rightarrow p^{\pm} + G^{n+} + n e^{-} \]

where $p^{\pm}$ is the incoming charged particle to be detected and $G$ a molecule of the gas. Most of the charge along a track, however, comes from secondary ionisation.

In contrast to primary ionisation, secondary ionisation can result from various different processes rather than one. For example, the electron from a primary ionisation can itself be energetic enough to ionise gas molecules. Such an electron is called a delta ray, or knock-on electron. Other processes involve excited gas molecules from interactions with the incoming particle:

\[ p^{\pm} + G \rightarrow p^{\pm} + G^{n+} + n e^{-} \]

Charged excited states can subsequently decay by the Auger effect in which the rearrangement of electrons inside an atom is accompanied by the ejection of an electron. Another possibility is for the incoming charged particle to create a neutral excited gas molecule, which de-excites by colliding with another type of gas molecule, ionising that molecule. This is called the Penning effect.

Lastly, I will mention one other mechanism that can produce secondary ionisation. An electron avalanche produces a comparable number of ionisations and photons. On average, these photons will travel further than delta rays or excited gas molecules. Some fraction of them will be energetic enough to produce additional ionisations, and therefore additional ionisations may happen relatively far away from the avalanche that produced the photon. The Geiger-Müller tube relies on this effect to operate.

| Table 1.1: Various processes that produce secondary ionisation in a gaseous detector |
|---------------------------------|-----------------|
| Delta ray                       | $e^{-} + G \rightarrow e^{-} + G^{+} + e^{-}$ |
| Auger effect                    | $G^{++} \rightarrow G^{2+} + e^{-}$ |
| Penning effect                  | $G_{1}^{+} + G_{2} \rightarrow G_{1} + G_{2}^{+} + e^{-}$ |
| Geiger-Müller effect            | $G + \gamma \rightarrow G^{+} + e^{-}$ |
1.2 Electron avalanche

After the primary and secondary electrons enter the amplification field they gain enough energy between collisions to further ionise or excite the gas molecules. In turn, the additional electrons will produce even more ionisations. This process continues and the amount of electron-ion pairs grows until all the electrons reach the anode. The amount of electrons resulting from a single initial electron is specified as the gas gain. Typical values are $10^3$–$10^5$ [2].

The gas gain fluctuates from event to event. This is a complicated process. It suffices to say that a Polya distribution is frequently used to describe the gain distribution [1]:

$$P(n; \lambda, \theta) = \frac{1}{\lambda} \left( \frac{\theta + 1}{\Gamma(\theta + 1)} \right) \left( \frac{n}{\lambda} \right)^{\theta} e^{-\frac{(\theta+1)n}{\lambda}}$$

Here $\lambda$ is the mean gain and $\theta$ is a positive real number to be determined empirically. The variance of this distribution is

$$\sigma^2 = \frac{\lambda^2}{\theta + 1}$$

The distribution is exponential for $\theta = 0$ and becomes Gaussian for large $\theta$. Figure 1.1 shows some examples of the Polya distribution for various values of $\theta$.

![Figure 1.1](image-url)  
**Figure 1.1:** The Polya distribution for various values of $\theta$. The distribution is exponential for $\theta = 0$ and becomes Gaussian for large $\theta$.

As mentioned above, photons may ionise the gas far away from an avalanche. For example, they may ionise the gas in the drift region and produce another avalanche. This may lead to a detector breakdown if, on average, one avalanche induces more than one additional avalanche.

To prevent this breakdown, a quench gas is added to the gas mixture. These typically consist of organic molecules which have absorption bands in the UV range. They reduce the distance that the photons travel before being absorbed. They can also absorb excess energy of excited gas molecules before they emit a photon.

1.3 Micro-pattern gaseous detectors

Gaseous detectors that are built on a micrometer scale constitute the class of micro-pattern gaseous detectors (MPGDs). Two well known types of MPGDs are Gaseous Electron Multipliers (GEMs) [3] and Micromegas [4] detectors. I will briefly describe these detectors in order to understand how GridPix relates to them.
In a GEM, three different electric fields can be identified: two drift fields and an amplification field. The first drift field transports the initial electrons to a so-called GEM foil (figure 1.2a) which consists of a thin (50–70 µm) polyimide film with a layer of copper on both sides. The foil contains holes that are typically 30–50 µm in diameter and are placed in a regular grid. By applying a potential difference between the two copper layers, an amplification field is produced in the holes. A potential difference between the pixel chip and the GEM foil produces the second drift field which transports the avalanche electrons to the pixel plane.

In Micromegas detectors, a metal mesh (figure 1.2b) and the pixel plane form the amplification field. The distance between the mesh and pixel plane is typically 25–150 µm. Insulating spacers prevent the mesh from being pulled towards the pixel plane by the electrostatic force. Contrary to GEMs, there is no second drift field that transports the avalanche electrons to the readout channels.

Figure 1.3 shows a picture of a GridPix on a Timepix chip. The working principle of GridPix is very similar to Micromegas detectors. The important difference is that, in GridPix the micro-mesh is built directly on top of the pixel chip using wafer post-processing techniques. This eliminates the problem of mesh misalignment associated with Micromegas detectors, which leads to moiré patterns in the efficiency spatial distribution. Also the support pillars in Micromegas give dead areas, whereas in GridPix they are precisely positioned between pixel readout pads.

A GridPix can be used with a large range of drift lengths above the grid. A drift region as short as 1 mm gives a so called Gossip detector, which was once proposed for the Atlas pixel-detector replacement. Intermediate drift lengths on the order of 1 cm will give a good track segment measurement. Longer drift lengths of a few meters can be used in time projection chambers as proposed for the International Linear Detector.

Figure 1.2: Picture of a GEM foil (a) and the metal mesh of a Micromegas with an insulating pillar (b).
Figure 1.3: Picture of a GridPix on Timepix taken with a scanning electron microscope. The grid is partially removed.
2 Modelling GridPix

This section describes how we model GridPix for our simulations. First, we will look at the gas region above the chip for which we will need to calculate the electric fields. After this, we make an approximate model of Timepix3. And lastly, we discuss the gas that we will use in our simulations.

2.1 Amplification gap and drift region

Figure 2.1 shows the geometry and approximate electric field strengths of the amplification gap and drift region. The electric field will be uniform in places sufficiently high above the grid. This can be used to reduce the amount of finite elements in simulations by lowering the cathode. For electric field calculations, we place it 60 µm above the grid. Later, when simulating drifting electrons or ions, we can choose the drift length to be as long as we want by defining the electric field to be uniform above this virtual cathode.

The pixel plane is covered with a layer of silicon-rich silicon-nitride (SiRN) which protects the electronics against discharges [5]. This material is slightly conductive and, therefore, can be defined as an equipotential when calculating the static electric field. Barring static leakage currents, it will be at the same potential as the pixel pads—close to 0 V.

This means that the static electric field above the protection layer is independent of the pixel chip below. Because of this, we can use the same amplification and drift fields for different pixel pad designs.

![Figure 2.1: Cross section of the Amplification gap and drift region modelled. Above the virtual cathode shown, the drift field is uniform and does not need to be meshed.](image)

2.2 Pixel plane

Moving charges in the detector induce signals on the pixel pads. To calculate these signals we need to model the pixel chip. Figure 2.2 shows a top view of two pixels in the Timepix3 chip. Only the top 4 metal layers are shown. It also defines a cutting plane, with cross section shown in figure 2.3. This cross section shows how the pixel pad is connected via the top 4 metal layers and it also includes the passivation layer, SiRN protection layer, gas volume, and substrate. However, it does not include the so-called 1× wiring levels right above the substrate. We will approximate the Timepix3 chip with the geometry shown in figure 2.4.
**Figure 2.2:** Top view of two pixels in a double column of the Timepix3 chip. Only the top 4 metal layers and substrate are visible. It makes for a good, albeit short, game of spot the difference. Figure produced using the top metal layer masks [6].

**Figure 2.3:** Cross section of two pixels at the cutting plane defined in figure 2.2. In reality, the SiRN protection layer is not flat and follows the contours of the chip surface. For simplicity, however, we will model it as being flat. The wiring levels that make up the front-end electronics are not shown. Figure produced using the top metal layer masks [6].
To model enlarged pixel pads, we will approximate the top metal layer with a continuous ground plane. This plane is covered with an oxide/nitride passivation layer (just like the Timepix3 chip). The enlarged pixel pad is located on top of the passivation layer and is covered with a protection layer. This model is shown in figure 2.5.

2.3 Gas

The simulations are based on a gas mixture of CO$_2$:DME (50:50) because this best fits other projects on GridPix in our group. It has very low diffusion: The minimum occurs at a drift field of 2 kV cm$^{-1}$ and is about 30 µm over a 1 mm distance [7].

In order to simulate drifting ions, Garfield++ requires us to define the ion mobility in the gas. However, relatively little is known about this and we have to make do with the ion mobilities in table 2.1 of CO$_2^+$ and DME$^+$ in their own gas. We use Blanc’s law to find the ion mobility $\mu$ in a gas mixture:

$$\frac{1}{\mu} = \sum_n f_n \mu_n$$

where $\mu_n$ are the pure ion mobilities, and $f_n$ the fractional gas concentrations. For our mixture, CO$_2$:DME (50:50), we find $\mu = 0.74$ cm$^2$ V$^{-1}$ s$^{-1}$. This is the best we can do for now, and we have to keep in mind how this uncertainty might affect our results.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Fraction</th>
<th>$\mu$ [cm$^2$ V$^{-1}$ s$^{-1}$]</th>
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<td>CO$_2$</td>
<td>0.5</td>
<td>1.09</td>
</tr>
<tr>
<td>DME</td>
<td>0.5</td>
<td>0.56</td>
</tr>
<tr>
<td>Total</td>
<td>1.0</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Figure 2.4: Top view (left) and cross section (right) of the model used to calculate the weighting field of a pixel pad in a Timepix3 chip. The top view only shows the aluminium and substrate.

Figure 2.5: Top view (left) and cross section (right) of the model used to calculate the weighting field of an enlarged pixel pad on top of a Timepix3 chip. The top view only shows the aluminium layers.
3 Charge collection

3.1 Signal induction

Moving charges inside a detector induce current signals on the readout electrodes. We can calculate these currents using the Shockley-Ramo theorem \[9,10\]:

\[ I_s = -q \psi \cdot v \]

where \( q \) is the charge, \( \psi \) the weighting field of the electrode, and \( v \) the velocity of the moving charge.

To find the weighting field of an electrode, we first set it to 1 V and all other electrodes to ground. Then we normalise the resulting electric field \( E_w \) to the electrode potential:

\[ \psi = \frac{E_w}{1V} \]

The net charge induced on an electrode in a time interval \( t_0 < t < t_1 \) is just the integrated current:

\[ Q_s = \int_{t_0}^{t_1} dt \, I_s = -q \int_{t_0}^{t_1} dt \, \psi \cdot v \]

But this is just the line integral of \( \psi \) along a curve \( \ell \) that has the parametrisation \( r(t_0 < t < t_1) \) with \( v = \dot{r} \). We can use the gradient theorem by writing the weighting field in terms of its potential \( \psi = -\nabla \phi \). The net charge now becomes

\[ Q_s = -q \int_{\ell} dr \cdot \psi = q[\phi(r_1) - \phi(r_0)] \]

3.2 Charge collection efficiency

In order to define the charge collection efficiency, consider the example in figure 3.1. It shows a parallel plate geometry and the weighting field of an 18 \( \mu \)m diameter disk in the bottom plate. Now suppose that during an electron avalanche the gas is ionised at \( r_0 \). The freed electron moves down to the anode and may further ionise the gas. Meanwhile, the ion moves up to the cathode. The electron moves much faster compared to the ion, and in fact we will consider the ion to be stationary during the development of an avalanche. After the electron stops moving the total signal charge is entirely due to the electron:

\[ Q_s^- = [\phi(r_-) - \phi(r_0)] \, e^- \] (3.1)

where \( r_- \) is the endpoint of the electron. Now the ion moves up and assuming its endpoint \( r_+ \) lies on an electrode, we find that \( \phi(r_+) = 0 \). So, the additional signal charge induced by the ion is

\[ Q_s^+ = [\phi(r_+) - \phi(r_0)] \, e^+ = \phi(r_0) \, e^- \] (3.2)

After the ions have finished drifting, the total signal charge becomes

\[ Q_s = Q_s^+ + Q_s^- = \phi(r_-) \, e^- \]

In other words, the total signal is completely determined by the electron’s endpoint.

If we apply equations 3.1 and 3.2 to an entire avalanche, then we find that the fast initial electron signal is

\[ Q_s^- = \sum_{i=1}^{N} [\phi(r_{-i}) - \phi(r_{0,i})] \, e^- = N[\bar{\phi}_- - \bar{\phi}_0] \, e^- \]
Figure 3.1: Electron ion pair moving in the weighting field of an 18 µm diameter disk in a parallel plate capacitor. Solid lines are equipotentials of the weighting potential, and dashed lines are field lines of the weighting field.

where $N$ is the gas gain, and $\bar{\phi}_-$ and $\bar{\phi}_0$ are the mean weighting potentials at the electron endpoints and ionisation positions respectively. In the same manner, the secondary slow ion signal is

$$Q_s^+ = \sum_{i=1}^{N} \phi(r_{0,i}) e^- = N\bar{\phi}_0 e^-$$

Again, the total signal charge is entirely determined by the electron endpoints:

$$Q_s = Q_s^+ + Q_s^- = N\bar{\phi}_- e^-$$

If all electrons would end up on the readout pad, then $\bar{\phi}_- = 1$ and $Q_s = Ne^-$. Accordingly, we define the charge collection efficiency as

$$\eta_{cc} = \frac{Q_s}{Ne^-} = \bar{\phi}_- \quad (3.3)$$

Furthermore, we define the fraction of the signal that is due to electrons as

$$r_- = \frac{Q_s^-}{Q_s} = 1 - \frac{\bar{\phi}_0}{\bar{\phi}_-}$$

and the fraction due to ions as

$$r_+ = \frac{Q_s^+}{Q_s} = \frac{\bar{\phi}_0}{\bar{\phi}_-} \quad (3.4)$$
4 Signal processing

The electron avalanche induces a signal on the readout electrodes. The analog front-end of a detector measures this signal. In this section I will briefly describe the front-end in a Timepix3 chip. For our current discussion it suffices to approximate the induced signal as

$$I_s(t) = Q_s^- \delta(t) + Q_s^+ \frac{e^{-t/\tau}}{\tau} H(t)$$  \hspace{1cm} (4.1)

Here $Q_s^-$ and $Q_s^+$ are the total charges induced by the electrons and ions respectively. The time constant $\tau$ depends on the ion mobility, the electric field strength, and the detector geometry. Here we will use the approximations $Q_s^+ : Q_s^- = 2 : 1$ and $\tau = 20 \text{ ns}$, which are based on the simulation results presented in section 8. Lastly, $\delta(t)$ is the Dirac delta function and $H(t)$ is the Heaviside step function, ensuring that the signal arrives after $t = 0$.

4.1 Preamplifier

Timepix3 has a charge sensitive amplifier (CSA) that is based on the Krummenacher scheme [11]. It provides a constant current discharge when the preamplifier output becomes large enough. Figure 4.1 shows a CSA with constant current discharge. Simply put, the input signal is integrated on a feedback capacitor $C_f$ and the output $V_{\text{out}}$ is proportional to the charge on $C_f$. Then the feedback capacitor is discharged by a current $\frac{1}{2} I_k$. For the signal in equation 4.1 the ideal response would be

$$V_{\text{out}}(t) = -\frac{1}{C_f} \int_0^t dt' \left( I_s(t') \pm \frac{1}{2} I_k \right)$$

where the sign of the discharge current depends on the signal polarity.

![Figure 4.1: A charge sensitive amplifier with constant current feedback. The sign of the discharge current is controlled by the preamplifier output.](image)

In reality the base amplifier has a limited bandwidth. As a result, it takes some time for the preamplifier to respond to the input signal. Additionally, the discharge current only reaches its maximum value if the output voltage becomes large enough.

Appendix A describes a model of the analog front-end that takes these effects into account. The goal of developing this model is to have a complete GridPix simulation. For example, we could generate initial ionisation along tracks using Heed [12] and simulate events (sets of values for column, row, ToA, and ToT). However, this is work in progress and not used. Figure 4.2 shows the response from this model. It also shows what the preamplifier output looks like with a noise of 75 $e^-$. 

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Figure 4.2: Response of a charge sensitive amplifier with constant current discharge for avalanche signals of various sizes. The left and right figures show the response with and without $75 e^-$ noise.

4.2 Discriminator

To detect and record events, the preamplifier output $V_{\text{out}}$ is fed into a discriminator which compares it to a threshold value. If $V_{\text{out}}$ exceeds this threshold, then the discriminator output goes high which tells the digital front-end to record an event.

It will store which pixel was hit, the time at which the discriminator fired (time of arrival, ToA), and also how long the signal remained above threshold (time over threshold, ToT).

4.3 Timewalk

Ideally, the ToA would be the time at which the signal arrives. However, the time at which the preamplifier output crosses the threshold value depends on the signal size and varies with gain fluctuations. This effect is called timewalk and is illustrated in figure 4.3. It shows how the rising edge of the preamplifier output affects the ToA measurement. In this example, the signal arrives at $t = 10$ ns. However, the recorded ToA is somewhat later than this because of the preamplifier's bandwidth and the slow ion component of the input signal. In this example the measured ToA for the $750 e^-$ signal is 33 ns, which means that there is a timewalk of 23 ns.

Another important mechanism that effects timewalk is propagation delay in the discriminator. In Timepix3 the discriminator consists of two amplification stages followed by a CMOS logic inverter. Each of these steps contributes to the overall propagation delay. This is outside the scope of this work, but a general description can be found in [13].

A method to correct a signal’s ToA for timewalk is based on the ToT. First, a general relationship between timewalk and ToT is determined empirically. This relationship is then used to correct the ToA of each event by subtracting the timewalk inferred from the ToT.
Figure 4.3: Preamplifier response for signals ranging from $750\,e^{-}$ to $2\,ke^{-}$ in steps of $250\,e^{-}$. The gray line indicates a $500\,e^{-}$ threshold. The right figure is an enlargement of the highlighted region in the left figure. It indicates the timewalk, time of arrival (ToA), and time over threshold (ToT) for the smallest signal.

4.4 Noise

The noise on the preamplifier output increases for a larger detector capacitance. Figure 4.4 shows this dependence for two values of the Krummenacher current $I_K$. For a bare Timepix3 chip, the detector capacitance is about $11\,fF$ [14].

Figure 4.4: Simulated equivalent noise charge as a function of detector capacitance for Krummenacher currents of $2.5\,nA$, and $12\,nA$. Data from [15].

For enlarged pads, the additional capacitance is dominated by the capacitance between the pad and the top-metal layer. We can estimate this as

$$\Delta C_D \approx A \left( \frac{d_{\text{oxygen}}}{\varepsilon_{\text{oxygen}}} + \frac{d_{\text{nitride}}}{\varepsilon_{\text{nitride}}} \right)^{-1}$$

The passivation consists of a silicon-oxide layer and a silicon-nitride layer. Their thicknesses are $d_{\text{oxygen}} = 1.35\,\mu m$ and $d_{\text{nitride}} = 0.45\,\mu m$, and their permittivities are $\varepsilon_{\text{oxygen}} = 3.9\,\varepsilon_0$ and $\varepsilon_{\text{nitride}} = 7\,\varepsilon_0$. Lastly, $A$ is the area of the hatched region in figure 4.5. For a $50\,\mu m$ diameter pad, we have $A = 735\,\mu m^2$. So, we find that the detector capacitance increases by about $16\,fF$ and that the equivalent noise charge increases by about $8\,e^{-}$. 

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Figure 4.5: Top view of a 50 µm diameter enlarged pixel pad and the top-metal layer
5 Protection layer

The pixel plane is covered with a resistive layer of silicon-rich silicon nitride (SiRN) which protects the electronics against discharges. The resistivity of this material is somewhere in the range of $10^9$–$10^{13}$ Ωm [16, 17]. Since it is slightly conductive, it requires special attention when calculating the signals induced by avalanches. These signals are an important part of my research, and therefore it is necessary to investigate what happens to charge on the protection layer.

In order to do so, I will first derive an autonomous differential equation that will provide insight into the time behaviour of free charge in matter. Secondly, we will consider an example that resembles the amplification gap and should illustrate what happens that arrives on the protection layer. After doing so, I can explain (i) how we should calculate the weighting field of the pixel pad, and (ii) the conditions for which my simulations will be applicable. More specifically, I will show that the resistivity of SiRN is sufficiently large to regard the protection layer as an insulator in signal calculations.

Finally, we will examine how charge buildup affects the gas gain. Charge on the protection layer will decrease the electric field and, in turn, this will decrease the gain. As a result, the protection layer will limit the rate of initial electrons that a pixel can tolerate.

5.1 Charge in matter

The current density of free charges $J_f$ in most materials is proportional to the force per unit charge. The constant of proportionality is the inverse resistivity (which is the conductivity), and the force is generally the Lorentz force [21]:

$$J_f = \frac{(E + v \times B)}{\rho}$$

Here $v$ is the velocity of the charges and $\rho$ the resistivity of the material. Usually, the velocity of the charges is small and we can ignore the second term. We can rewrite the equation in the form of Ohm’s law:

$$E = J_f \rho$$

(5.1)

For simplicity I will assume the material to be isotropic and linear¹, but not necessarily homogeneous. In such a material, the electric field and displacement field are proportional to each other: $D = \epsilon E$, where $\epsilon$ is the permittivity of the material. Using this to rewrite equation 5.1 in terms of the displacement field gives

$$D = J_f \tau$$

(5.2)

where we have written $\tau \equiv \rho \epsilon$. Later, this will turn out to be the time constant for charge dissipation.

Local conservation of charge also tells us something about the current density: it should satisfy the continuity equation

$$\nabla \cdot J_f = -\dot{\rho}_f$$

(5.3)

¹In a linear medium, the polarisation density is proportional to the electric field: $P = \chi_e \epsilon_0 E$ where $\chi_e$ is the electric susceptibility of the medium and satisfies the equation $\epsilon = \epsilon_0(\chi_e + 1)$. 

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where \( \rho_f \) is the free charge density. Using equation 5.2, we rewrite the left-hand side as

\[
\nabla \cdot \mathbf{J}_f = \nabla \cdot \frac{\mathbf{D}}{\tau} = \frac{1}{\tau} \nabla \cdot \mathbf{D} - \frac{1}{\tau^2} \mathbf{D} \cdot \nabla \tau
\]

Now, we further rewrite the first term on the right-hand side using Gauss’s law \( \nabla \cdot \mathbf{D} = \rho_f \); and the second term using equation 5.2 again:

\[
\nabla \cdot \mathbf{J}_f = \frac{\rho_f - \mathbf{J}_f \cdot \nabla \tau}{\tau}
\]

Finally, plugging this back into equation 5.3 gives us the final autonomous differential equation for the time dependence of charge distribution in matter:

\[
\dot{\rho}_f \tau + \rho_f - \mathbf{J}_f \cdot \nabla \tau = 0 \tag{5.4}
\]

It is important to realise that we cannot use this equation with just any free charge distribution \( \rho_f \) if there are ideal conductors present. These put additional constraints on \( \rho_f \) that are not important here.

At any given time the free charge density \( \rho_f(t, \mathbf{r}) \) and permittivity \( \epsilon(\mathbf{r}) \) fields determine the electric field \( \mathbf{E}(\mathbf{r}) \). The resistivity \( \rho(\mathbf{r}) \) and \( \mathbf{E}(\mathbf{r}) \) determine the current density \( \mathbf{J}_f(t, \mathbf{r}) \); and the change in \( \rho_f(t, \mathbf{r}) \) is given by equation 5.4. Within a material of constant \( \tau \) we have \( \nabla \tau = 0 \) and equation 5.4 is easily solved:

\[
\mathbf{r} \in \{ \mathbf{r} | \nabla \tau(\mathbf{r}) = 0 \} \implies \rho_f(t, \mathbf{r}) = \rho_f(t_0, \mathbf{r}) \exp\left(-\frac{t - t_0}{\tau}\right)
\]

From this we conclude the following:

Regardless of the geometry, charge inside a resistive material of constant \( \rho \epsilon \) dissipates with a time constant \( \tau = \rho \epsilon \).

From Equation 5.4 we can also see that charge can only accumulate in places where \( \tau \) changes (\( \nabla \tau(\mathbf{r}) \neq 0 \)). By accumulate, I mean that \( \dot{\rho}_f \) has the same sign as \( \rho_f \), which means that the amount of charge \( |\rho_f| \) increases.

Although equation 5.4 provides insight into the behaviour of free charge in matter, it is not very practical to use when \( \tau \) varies discontinuously from one place to another. This is often the case when different materials are in contact with one another.

### 5.2 Charge dissipation

Now, we will consider the parallel plate capacitor in figure 5.1 in order to determine what happens to charge on the protection layer. The bottom plate contains circular pixel pads that are not in contact with the surrounding ground plane. Furthermore, the material between the two plates has resistivity \( \rho \) and permittivity \( \epsilon \).

Suppose that at \( t = 0 \) a uniform surface charge suddenly appears on top of the protection layer: \( \sigma(0) = Q_{\text{ava}}/p^2 \), where \( Q_{\text{ava}} \) is the charge from an avalanche and \( p \) the pixel pitch. The free charge induced on the bottom plate is \( -\sigma \). The charge seen by the preamplifier right after \( t = 0 \) is

\[
Q_s(0) = -\sigma(0) A_{\text{pad}} = \frac{A_{\text{pad}}}{p^2} Q_{\text{ava}}
\]

where \( A_{\text{pad}} \) is the pixel pad surface area.
Figure 5.1: Parallel plate capacitor with a resistive medium to illustrate how the charge density $\sigma$ on the protection layer depends on time. (This figure is not to scale.)

Immediately, the free charge starts moving under influence of the electric field inside the protection layer. The current density is

$$J_f = \frac{E}{\rho} = -\frac{\sigma}{\tau} \hat{e}_z$$

where $\tau \equiv \rho \epsilon$. The change in $\sigma$ is equal to the current flowing into the top of the protection layer:

$$\dot{\sigma} = J_f \cdot \hat{e}_z = -\frac{\sigma}{\tau}$$

and thus

$$\sigma(t) = \sigma(0) \exp\left(-\frac{t}{\tau}\right)$$

So far, we have ignored the passivation layer and all charge induced on the bottom plate can move freely through the protection layer. This means that $Q_s$ does not change any more. However, if we include the passivation layer, then any charge on the ground plane surrounding the pixel pad cannot move through the protection layer. However, $\sigma$ should become zero after waiting long enough: $\sigma(t \gg \tau) = 0$. Since it must have gone into the pixel pads, the charge seen by the preamplifier will be

$$Q_s(t \gg \tau) = Q_{ava}$$

If the charge seen by the preamplifier changes too quickly, the relationship between ToT and charge will not be linear any more. Assuming that, initially, the preamplifier sees a charge of 6000 $e^-$ and that it has a constant discharge current of 1 nA, we find that the output of the preamplifier returns to baseline in about 1 $\mu$s. The characteristic time in which $Q_s$ changes should be much longer than this. Based on the example considered above, we expect this characteristic time to be of the same order of magnitude as $\rho \epsilon$. For SiRN, which has $\epsilon_r = 7$, we require that $\rho \gg 2 \times 10^4$ $\Omega \cdot$ m.\(^2\) If the resistivity satisfies this condition, then we can calculate the weighting field of a pixel pad by assuming that the protection layer is an insulator.

5.3 Charge buildup

In the previous section, we determined how charge from a single avalanche dissipates and found a minimum resistivity requirement for the protection layer. We will now examine what happens when charge continuously arrives at the protection layer by adding another term to equation 5.5:

$$\dot{\sigma} = -\frac{\sigma}{\tau} + \dot{\sigma}_{ava}$$

and find an upper limit to the resistivity.

\(^2\)Note that there are several other requirements on the resistivity that may set lower limits that are higher than the one presented here.
Suppose that the rate of initial electrons arriving at a pixel is \( f \) and the mean gain is \( N \). We will approximate the incoming surface charge density as

\[
\dot{\sigma}_{\text{ava}} = \frac{fN e^-}{p^2}
\]  

(5.7)

where \( e^- \) is the electron charge: \( e^- = -1.6021766208(98) \times 10^{-19} \text{C} \).

The mean gas gain in a uniform electric field is given by

\[ N = \exp(\alpha d) \]  

(5.8)

where \( \alpha \) is the Townsend coefficient and \( d \) the distance traversed by the avalanche. Figure 5.2 shows the dependence of the Townsend coefficient on the electric field in an even mixture of \( \text{CO}_2 \) and DME. For sufficiently high electric fields, \( \alpha \) is linear in \( E \):

\[ \alpha(E_0 + \Delta E) \approx \alpha(E_0) + \frac{d\alpha}{dE}_{E_0} \Delta E \equiv \alpha_0 + \alpha' \Delta E \]

with \( \alpha' = 0.02495(3) \text{ V}^{-1} \). Substituting this into equation 5.8 gives us the gain in terms of a change in the electric field. In our example we find that

\[ N = \exp[(\alpha_0 + \alpha' \Delta E)d_{\text{amp}}] = N_0 \exp(\alpha' \Delta E d_{\text{amp}}) \]  

(5.9)

where \( d_{\text{amp}} \) is the height of the amplification gap and \( N_0 \) is the unperturbed gain before there is any change in the electric field.

**Figure 5.2:** Townsend coefficient \( \alpha \) at various electric fields in \( \text{CO}_2\):DME (50:50) as computed by Magboltz [22]. In sufficiently high fields, \( \alpha \) is linear in \( E \) and we find \( \frac{d\alpha}{dE} = 0.02495(3) \text{ V}^{-1} \).

Charge accumulating on top of the protection layer will change its potential by

\[ \Delta V = \frac{d_{\text{prot}} \sigma}{\epsilon} \]

As a result, the electric field in the amplification gap changes by

\[ \Delta E = \frac{\Delta V}{d_{\text{amp}}} = \frac{d_{\text{prot}} \sigma}{d_{\text{amp}} \epsilon} \]  

(5.10)
Substituting equations 5.7, 5.9, and 5.10 into equation 5.6 gives us

\[ \dot{\sigma} = -\frac{\sigma}{\tau} + \frac{f N_0 e^{-}}{p^2} \exp\left(\frac{\alpha' \delta_{prot} \sigma}{\epsilon}\right) \]  

(5.11)

After some time, this system will reach a steady state: the rate of dissipation will be equal to the rate at which charge arrives on the protection layer. Solving equation 5.11 for \(\dot{\sigma} = 0\), and plugging in the definition of \(\tau\) gives us

\[ \sigma = -\frac{\epsilon}{\delta_{prot} \alpha'} W_p\left(-\frac{f \rho N_0 \delta_{prot} \alpha' e^{-}}{p^2}\right) \]  

(5.12)

Here, \(W_p(x)\) is the principal branch of the Lambert W function, or product logarithm. It is defined by

\[ W_p(x) \exp(W_p(x)) \equiv x \quad \text{for} \quad x > -\exp(-1) \]

Figure 5.3 shows the two branches of the Lambert W function.

\[ \text{Figure 5.3: The two branches } W_p \text{ and } W_m \text{ of the Lambert W function} \]

For conciseness, we write

\[ \chi \equiv -\frac{f \rho N_0 \delta_{prot} \alpha' e^{-}}{p^2} \]

Now, we find the change in the electric field by substituting equation 5.12 into equation 5.10:

\[ \Delta E = -\frac{W_p(\chi)}{\alpha' d_{amp}} \]

Plugging this into equation 5.9 leads to the relative change in gain:

\[ \frac{N}{N_0} = \frac{W_p(\chi)}{\chi} \]  

(5.13)

Figure 5.4 shows the change in the electric field and the drop in gain for various resistivities and initial gains. The values of the parameters used in these examples are repeated in table 5.1.
Figure 5.4: Change in the electric field (top) and gain (bottom) versus the initial electron rate per pixel for initial gains ranging from 2500 to 12500 in steps of 2500. The lower axes specify the rate for $\rho = 10^9$ and $10^{13} \text{Ω m}$, and the top axes specify the product of $f$ and $\rho$. For example, if the minimum allowable relative gain $N_{\text{min}}/N_0$ is 85 % and the initial gain is 5000, then $f\rho$ must be less than $\sim 7 \times 10^{12} \text{ Hz Ω m}$. If we require the detector to operate with an initial electron rate of 10 kHz, then the resistivity needs to be less than $\sim 7 \times 10^8 \text{ Ω m}$.
Table 5.1: Parameters used in this section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
<td>$p$</td>
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</tr>
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</tr>
<tr>
<td>$d_{prot}$</td>
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</tr>
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<td>$\epsilon$</td>
<td>$7 \epsilon_0$</td>
</tr>
<tr>
<td>$\alpha'$</td>
<td>0.02495(3) $V^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$10^9$–$10^{13}$ Ω m</td>
</tr>
<tr>
<td>$N_0$</td>
<td>2500–12500</td>
</tr>
</tbody>
</table>

To conclude this section, I will explain how we can use equation 5.13 to calculate the rate of initial electrons $f$, the resistivity $\rho$, the initial gain $N_0$, or any of the other parameters in $\chi$ actually. Figure 5.5 show the drop in gain given by equation 5.13 as a function of $\chi$. In this figure we can see that a certain allowable drop in gain corresponds to some maximum value of $\chi$. More concretely, suppose that the minimum allowable gain is $N_{\text{min}}$. This means that we require

$$\frac{W_{\rho}(\chi)}{\chi} > \frac{N_{\text{min}}}{N_0} \iff \chi < -\frac{\log(N_{\text{min}}/N_0)}{N_{\text{min}}/N_0}$$

Less concisely:

$$-f\rho N_0 d_{prot} \alpha' e^- \frac{\epsilon}{p^2} < -\frac{\log(N_{\text{min}}/N_0)}{N_{\text{min}}/N_0}$$

which generally gives an upper limit to the resistivity.

For example, suppose that we want a GridPix to work in an environment where the initial electron rate will be $f$, and the minimum allowable relative gain is $N_{\text{min}}/N_0$. Furthermore, we may be free to choose the initial gain $N_0$ by setting the grid voltage, or we may try to tweak the resistivity $\rho$ and thickness $d_{prot}$ of the protection layer. In any case, we need to find a set of values for the free parameters that will satisfy this inequality in combination with all the fixed parameters.

![Figure 5.5: Relative gain as a function of $\chi$](image-url)
6 Electric field calculation

6.1 Meshing and finite element analysis

We calculate the electric fields using the finite element method. For this we use Elmer [23], which is open source software for solving multiphysical problems. It is developed mainly by CSC - IT Center for Science. We mesh the geometry with Gmsh [24], which is also open source software.

The Elmer class in Garfield++ requires us to mesh our models with **quadratic tetrahedrons**. This type of element consists of ten nodes: four corner nodes, and six nodes located on the edges connecting the corner nodes (figure 6.1a). They are more suitable than linear tetrahedrons for meshing geometries that contain curved surfaces (figure 6.1b).

![Figure 6.1: A quadratic tetrahedron (a) and an example showing how quadratic triangles can approximate circular arcs (b)](image)

6.2 Drift and amplification field

As I mentioned in section 2.1, the top of the protection layer will be at 0 V because it is slightly conductive and connected to the pixel pads. The grid and virtual cathode will be at $-600$ V and $-606$ V respectively. These are the Dirichlet boundary conditions in the electric field calculation.

For simplicity, we restrict ourselves to pixels in the centre of the chip. The field in pixels sufficiently far from the chip edge can be calculated by considering an infinite grid of pixels. The mirror symmetries of this infinite grid define Neumann boundary conditions. To see how, suppose that $\mathbf{n}$ is the normal vector to a plane of mirror symmetry. The electric field on this plane must satisfy $\mathbf{E} \cdot \mathbf{n} = 0$, or in terms of its potential:

$$\nabla_n V = 0$$

where $\nabla_n$ is the directional derivative $\mathbf{n} \cdot \nabla$. This is the boundary condition on the planes of mirror symmetry shown in figure 6.2.

As a result, we only need to calculate the electric field in a triangular region that makes up $1/8$ of a pixel cell. Figure 6.3 shows the result of the electric field calculation.
Figure 6.2: Schematic top view of a GridPix. The mirror symmetries define Neumann boundary conditions. We only need to calculate the electric field in the shaded triangular region.

Figure 6.3: The electric field in a GridPix with Field lines (red) and equipotentials (green)
6.3 Weighting field

We approximate the weighting field by making a grid of $5 \times 5$ pixels and setting the centre pad to 1 V and the outer sides to 0 V. Figure 6.4 schematically shows the boundary conditions on the side planes of the geometry. We need to calculate the field in a rectangular region for the Timepix3 model, and in a triangular region for enlarged pixel pads.

![Figure 6.4](image)

**Figure 6.4:** Schematic top view of the pixel plane showing the mirror symmetries in the model of a Timepix3 chip (a) and the model of enlarged pixel pads (b). The planes far away from the centre pixel are approximately at 0 V.

Figure 6.5 shows the weighting potential for Timepix3 and an enlarged pixel pad that is $50 \mu m$ in diameter. Considering equation 3.3, the most important thing to notice is that the enlarged pad significantly increases the weighting potential on the protection layer (at $z = 0 \mu m$). Also note that the weighting potential is less than about $10^{-2}$ above the grid. If the fraction of ions drifting back into the drift region is about 10%, then they only contribute about 0.1% of the total integrated signal.

6.4 Expected charge collection efficiency

To calculate how the charge collection efficiency depends on the pixel pad diameter, we approximate the distribution of electron endpoints, $f_-$, as a bivariate Gaussian distribution that is rotationally symmetric:

$$f_-(r) = \frac{1}{2\pi \sigma^2} e^{-r^2/2\sigma^2}$$  \hspace{1cm} (6.1)

where $r^2 = x^2 + y^2$ and $\sigma^2$ is the variance in both the $x$ and $y$ directions. The radial distribution of electron endpoints is

$$\int_0^{2\pi} d\theta \, r \, f_-(r) = 2\pi r \, f_-(r)$$  \hspace{1cm} (6.2)

and is shown in figure 6.6 for various values of $\sigma$.

The charge collection efficiency for the electron distribution from equation 6.1 is

$$\eta_{kc} = \bar{\phi}_- = \int_0^\infty dr \int_0^{2\pi} d\theta \, r \, f_-(r) \, \phi(r)$$  \hspace{1cm} (6.3)

where $r = r(\cos \theta, \sin \theta, 0)$. Figure 6.8 shows the result of our estimate of the charge collection efficiency as a function of pixel pad diameter for various values of $\sigma$. 
Figure 6.5: The weighting field of a pixel pad in the Timepix3 model (top) and for enlarged pixel pads (bottom), with red field lines and green equipotentials on a logarithmic scale. Note that the weighting potential is less than about $10^{-2}$ above the grid.
Figure 6.6: Approximation of the radial distribution of electron endpoints on the protection layer

Figure 6.7: The weighting potential on the protection layer ($z = 0 \, \mu m$), where avalanche electrons arrive, for different pixel pad diameters. The vertical gray lines indicate the pixel cell boundaries in the $x$ and $y$ directions (left), and the diagonal direction (right).

Figure 6.8: The expected dependence of the charge collection efficiency on the pixel pad diameter
The variance of the electron distribution from equation 6.1 can be calculated as

$$\sigma^2 = D_t^2 d_{\text{amp}}$$

where $D_t$ is the transverse diffusion coefficient and $d_{\text{amp}}$ is the height of the amplification gap. For CO$_2$-DME (50:50) at a field strength of 100 kV cm$^{-1}$, the transverse diffusion coefficient calculated by Magboltz [22] is 0.930(11) $\sqrt{\text{µm}}$. For an amplification gap of $d_{\text{amp}} = 60$ µm, we find $\sigma = 7.21(9)$ µm. Since there is also some spread in the position at which the initial electron crosses the grid hole, we expect the actual value of $\sigma$ to be slightly higher than this.

**Grid height variation**

In [7] it is shown that the grid height has a spread of about 3 µm over a single wafer. The diffusion coefficient depends on the electric field strength, and therefore this variation in grid height can affect the charge collection efficiency. To investigate this, we will consider variation in grid height among chips (inter-chip variation) and variation in grid height within a single chip (intra-chip variation). They lead to different changes in the electric field strength: inter-chip variation can be compensated for by choosing a different grid voltage to obtain the required gas-gain, but intra-chip variation cannot be compensated for—there is only one grid voltage on single chip.

We will first consider inter-chip variation by considering its effect on the gain using equation 5.9:

$$N = \exp\left((\alpha_0 + \alpha' \Delta E)(d_{\text{amp},0} + \Delta d_{\text{amp}})\right)$$

$$= N_0^{1+\Delta d_{\text{amp}}/d_{\text{amp},0}} \exp[\alpha' \Delta E (d_{\text{amp},0} + \Delta d_{\text{amp}})]$$

(6.4)

Requiring that $N = N_0$ leads to

$$\Delta E = -\frac{\Delta d_{\text{amp}} \log(N_0)}{\alpha' d_{\text{amp},0} (d_{\text{amp},0} + \Delta d_{\text{amp}})}$$

(6.5)

For intra-chip variation, on the other hand, we keep the grid voltage constant and find

$$\Delta E = \frac{V}{d_{\text{amp},0} + \Delta d_{\text{amp}}} - \frac{V}{d_{\text{amp},0}}$$

$$= -E_0 \frac{\Delta d_{\text{amp}}}{d_{\text{amp},0} + \Delta d_{\text{amp}}}$$

(6.6)

Figure 6.9 shows the transverse diffusion coefficient as a function of electric field strength in CO$_2$:DME (50:50). We will approximate it as

$$D_t \approx D_{t,0} + D_{t}' \Delta E$$

and

$$\sigma^2 \approx \left(\frac{\sigma_0}{\sqrt{d_{\text{amp},0}}} + D_{t}' \Delta E\right)^2 (d_{\text{amp},0} + \Delta d_{\text{amp}})$$

(6.7)

Figure 6.10 shows how both the electric field and the standard deviation of the electron distribution change as a function of an error in the grid height according to equations 6.5, 6.6, and 6.7. In figure 6.11 we show how an error in the grid height affects the charge collection efficiency as calculated in equation 6.3 for pixel pad diameters of 20 µm and 50 µm. The change in charge collection efficiency is approximately proportional to the error in grid height. In figure 6.12 we show the factor of proportionality as a function of pixel pad diameter. Table 6.1 lists the values for all the relevant parameters used in these figures.
**Figure 6.9:** The transverse diffusion coefficient as computed by Magboltz and a linear straight line fit with $\frac{dD_t}{dE} = -0.00360(9) \sqrt{\mu m} (kV cm^{-1})^{-1}$

**Table 6.1:** Parameters used in this section

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
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<td>$\alpha'$</td>
<td>2.495(3) $\times 10^{-2}$ V$^{-1}$</td>
</tr>
<tr>
<td>$D'_t$</td>
<td>3.60(9) $\times 10^{-3}$ $\sqrt{\mu m} (kV cm^{-1})^{-1}$</td>
</tr>
</tbody>
</table>
Figure 6.10: Change in electric field $\Delta E$ below the grid (left) and in standard deviation $\sigma$ of the electron distribution on the protection layer (right) as a function of error in grid height $\Delta d_{\text{amp}}$. The solid curve is for inter-chip variation in the grid height, which can be compensated for by adjusting the grid voltage, and the dashed curve is for intra-chip variation in the grid height.

Figure 6.11: Change the charge collection efficiency $\eta_{\text{cc}}$ for a 20$\mu$m diameter pixel pad (left) and a 50$\mu$m diameter pixel pad (right) as a function of error in grid height $\Delta d_{\text{amp}}$. The solid curve is for inter-chip variation in the grid height, and the dashed curve is for intra-chip variation in the grid height.

Figure 6.12: Ratio of the uncertainty in the charge collection efficiency $\eta_{\text{cc}}$ to the uncertainty in the grid height $d_{\text{amp}}$
7 Simulations

We will simulate avalanches in Garfield++ using the AvalancheMicroscopic class. This class is an implementation of the methods presented by Schindler in [25]. Here it suffices to say that this class will simulate how electrons drift through the detector and how they ionise the gas.

If we want to simulate the time dependent signals, then we also need to simulate the drift of ions. For this we use the AvalancheMC class (where MC stands for Monte Carlo), because microscopic tracking is only available for electrons.

Next, we need to determine where to initiate the avalanches. Currently, we are not interested in simulating tracks—in other words, we do not care where the initial electrons come from. Ideally, we want them uniformly distributed throughout the gas volume. The drift height is typically much larger than the height of our virtual cathode. Therefore, ionisations below the virtual cathode only make up a tiny fraction of all events (1.2% for a 1 cm drift height). The electric field above the virtual cathode is uniform, and as a result electrons will arrive uniformly distributed at the $z = 121 \mu m$ plane. Accordingly, we will initiate avalanches by releasing electrons uniformly distributed on this plane above the centre pixel: $|x| < \frac{1}{2}p$, $|y| < \frac{1}{2}p$, and $z = 121 \mu m$. Here $p$ is the pixel pitch.

Occasionally, an electron will drift to a neighbouring pixel. When this happens, we translate the entire event back to the centre pixel (see figure 7.1). This is equivalent to initiating avalanches in the entire $z = 121 \mu m$ plane and only looking at events in the centre pixel.

![Figure 7.1: Some electrons arrive at neighbouring pixels and are translated back to the centre pixel (arrows)](image)

7.1 Charge collection efficiency

With the simulations as described above, we will measure the charge collection efficiency $\eta_{cc}$ for Timepix3 and for enlarged pixel pads of various diameters. We calculate $\eta_{cc}$ as explained in section 3.2: we determine the electron endpoints for each avalanche and calculate the mean weighting potential at these points using the weighting fields described in section 6.3.

7.2 Detection efficiency

Each pixel pad is connected to a preamplifier which amplifies the signal induced by an electron avalanche. The output of this preamplifier is converted to a digital signal by a discriminator. Its output will be high if the preamplifier output is above a certain voltage threshold.

Determining whether or not the discriminator will “fire” for a given signal is not straightforward and depends on the design of the preamplifier and the discriminator. If, however, the
shape of a signal that contains some charge $Q_s$ does not vary too much from event to event, then a voltage threshold $V_{\text{thr}}$ will correspond to some charge threshold $Q_{\text{thr}}$.

We will measure the detection efficiency for each pixel plane model by determining the fraction of events that satisfy

$$N\eta e^- > Q_{\text{thr}}$$

where $N$ is the gain.

### 7.3 Crosstalk

Figure 6.7 shows that enlarging a pixel pad will also increase its weighting potential in neighbouring pixels. This might lead to crosstalk. Therefore, we will measure the charge collection “efficiency” of the pixel neighbouring the centre pixel. Applying a charge threshold, we will also measure the detection “efficiency” for an avalanche in a neighbouring pixel.
8 Results

We will first look at some general results of the simulations before moving on to the measurements outlined in the previous section. Note that the uncertainties reported here are the statistical uncertainties in our data, unless stated otherwise.

Figure 8.1 shows a histogram of the gain together with an unbinned maximum likelihood fit of a Polya distribution. We find that the mean gain in our simulations was $7416(17)$. The value of $\theta$ for which the Polya distribution is most likely to generate the gains that we observe is $0.573(6)$. Figure 8.2 shows how the gain depends on the radius at which the initial electron crosses the grid hole. The mean gain varies from about 6000 to 10000, whereas $\theta$ does not change. We also see that integrating the Townsend coefficient is a good way to calculate the gain in the centre of the grid hole. Near the edges, however, this method significantly overestimates the gain.

![Figure 8.1: Distribution of avalanches in gain. The red curve is an unbinned maximum likelihood fit of a Polya distribution.](image)

![Figure 8.2: Unbinned maximum likelihood fit results for the mean gain (left) and the parameter $\theta$ (right) for different radii at which the electrons crossed the grid hole. The dashed line is the gain calculated by integrating the Townsend coefficient along field lines crossing the grid hole at a radius $r$.](image)

Figure 8.3 shows the distribution of ionisations in height $z$ and distance from the centre $r$. The mean ionisation height is $6.2 \mu m$ above the protection layer. The number of ionisations decreases exponentially with increasing height up until the grid. Beyond this, the number of
ionisations decreases rapidly up to about \( z = 65\, \mu m \). Comparing this with the electric field in figure 6.3, we see that the electric field strength decreases rapidly above this height. The fraction of ionisations above the grid is \( 8.97(10) \times 10^{-6} \).

Figure 8.4 shows the distribution of electron endpoints on top of the protection layer. The distribution peaks at \( r = 8\, \mu m \) and remains above half its peak value until \( r = 15\, \mu m \). A maximum likelihood fit of the distribution in equation 6.2 yields \( \sigma = 8.03537(13)\, \mu m \). The distributions also show that relatively few avalanche electrons arrive in a neighbouring pixel cell: about 0.13\% (this does not include electrons that drifted to neighbouring pixels before crossing the grid).

In figure 8.5 we show a time dependent signal induced on a Timepix3 pixel pad by an avalanche with a gain of 9426. First, the electrons induce a short signal. This is followed by a long ion signal. Eventually, the integrated signal reaches a value of 3569 \( e^- \). This means that the charge collection efficiency for this particular avalanche was only 38\%.

Figure 8.6 shows the number of electrons and ions that have not yet arrived at the pixel plane or any other electrode such as the grid or cathode. It shows that electrons already start arriving at the pixel plane while the avalanche is still growing. It also shows that the number of moving ions decreases quickly between 60 ns and 75 ns. These are ions arriving at the grid. Some ions go through the grid hole, but not all of these ions drift up into the drift region. Most of them arrive on the top side of the grid. This takes a long time because these ions see a relatively weak electric field. The fraction of ions that do drift up into the drift region is 3.7\% for this avalanche.

Enlarging the pixel pad increases the fraction of the signal that is induced by ions (equation 3.4). Figure 8.7 shows this effect. Error bars indicate the 68.3\% central confidence interval of the underlying distributions.

### 8.1 Charge collection efficiency

Figure 8.8 shows the mean charge collection efficiency for various pixel pad diameters. Error bars indicate the 68.3\% central confidence interval of the underlying distributions instead of the uncertainty in the mean values. For example, for our Timepix3 model without enlarged pads, the charge collection efficiency lies in the range 46.93–60.88\% for 68.3\% of all avalanches and the mean value is 54.00(4)\%. For a 50\,\mu m diameter pad this interval is 96.03–97.35\%, and the mean is 96.632(4)\%. We also show the expected (mean) charge collection efficiency calculated with equation 6.3 using the result \( \sigma = 8.03537(13)\, \mu m \) from above (figure 8.4).

Also shown in figure 8.8 is the ratio of the uncertainty in the mean charge collection efficiency to the uncertainty in the grid height. This figure is produced in the same way as figure 6.12, but now with the parameters \( N_0 = 7416(17) \) and \( \sigma_0 = 8.03537(13)\, \mu m \) from the results above (figures 8.1 and 8.4).

Figure 8.9 shows that the charge collection efficiency strongly depends on the position and size of the electron endpoint distribution. Avalanches that are closer to the pixel centre induce a larger fraction of charge on the pad than avalanches that are further away. Avalanches that are more spread out over the pixel plane induce a smaller fraction of charge on the pad than avalanches that are concentrated more in a single spot.
Figure 8.3: Distribution of ionisations in height (left), and in height and radius (right). From the left histogram we can see that the number of ionisations per unit length decreases exponentially in $z$.

Figure 8.4: Distribution of electron endpoints in distance from the pixel centre (left) and a hit map of electrons at the protection layer (right). The most probable distance is about 8 $\mu$m from the pixel centre, while some arrive much further out. For clarity: from the left figure we read off that the number of electrons per unit area at $r = 8 \mu$m is about $3.4 \times 10^5/(2\pi \times 8 \times 0.05 \mu m^2) = 3.4 \times 10^5/(0.5 \times 0.5 \mu m^2)$. This is the same value that we would read off on the right plot at $x = 8 \mu$m and $y = 0 \mu$m for example.
Figure 8.5: Signal induced on the pixel pad of a Timepix3 chip by an avalanche with a gain of 9426. At $t = 0$ ns, the initial electron crosses the grid hole. The top and bottom graphs show the integrated signal $Q_s$ and the current signal $I_s \equiv \frac{dQ_s}{dt}$. The left and right graphs have time scales that are appropriate for showing the electron and ion contributions to the signal.

Figure 8.6: The number of moving electrons and ions as a function of time for the same avalanche as in figure 8.5. The number of ions at $t = 100$ ns is about $1.2 \times 10^3$. Most of these are ions that cross the grid at a radius $r > 4$ µm. They slowly turn around and eventually arrive on top of the grid. The actual ion backflow (ions not ending up on the grid) is about 4% for this event.
Figure 8.7: Fraction of the signal due to ions as a function of pad diameter. Error bars indicate the 68.3% central confidence interval of the underlying distributions.

Figure 8.8: Charge collection efficiency (left) and the uncertainty in the mean charge collection efficiency divided by the grid height uncertainty (right) as a function of the pixel pad size. Error bars indicate the 68.3% central confidence interval of the underlying distributions. Also shown is the expected charge collection efficiency from equation 6.3.
8.2 Detection efficiency

Figure 8.10 reports the detection efficiency for different pad sizes and charge thresholds. For a charge threshold of 500 $e^-$ the detection efficiency increases from 94.42(28) % for Timepix3 to 97.60(28) % for a 50 $\mu$m diameter pad. For higher thresholds the increase is larger: for a charge threshold of 1000 $e^-$ the detection efficiency increases from 86.58(27) % for Timepix3 to 93.84(28) % for a 50 $\mu$m diameter pad.

8.3 Crosstalk

In figure 8.11 we report the charge collection and detection “efficiencies” for the neighbouring pixel pad located at $x = -55 \mu$m and $y = 0 \mu$m. The fraction of collected charge increases for larger pads and also varies more from event to event. The fraction of events that exceed a certain threshold also increases with larger pad size. For pad diameters up to 30 $\mu$m, none of the events exceed the 500 $e^-$ threshold. For a 50 $\mu$m pad the fraction of events that exceed a 500 $e^-$ threshold is 0.107(9) %. Table 8.1 contains the same information as the detection efficiency in figure 8.11, but shows the actual number of events instead of fractions.

**Table 8.1:** Number of events above threshold at the neighbouring pixel with Poisson errors. The total number of events is 121000.

<table>
<thead>
<tr>
<th>$D$ [$\mu$m]</th>
<th>$Q_{\text{thr}}$ [$e^-$]</th>
<th>500</th>
<th>750</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td></td>
<td>2(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>19(4)</td>
<td>3(2)</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td></td>
<td>57(8)</td>
<td>15(4)</td>
<td>3(2)</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>130(11)</td>
<td>26(5)</td>
<td>12(3)</td>
</tr>
</tbody>
</table>

In figure 8.12 we show what the time dependent signal looks like in the neighbouring pixel. The electron signal now has the opposite sign and the ion signal is bipolar. The integrated signal first peaks at 17 ns with a value of $-117 e^-$. Eventually the signal reaches 60 $e^-$. The charge collection “efficiency” for this event is 0.64 %.
Figure 8.9: Scatter plot containing avalanches. The horizontal axis shows the distance of the mean electron endpoint to the pixel centre. The vertical axis shows the electron endpoint variances in $x$ and $y$ added in quadrature. Colour shows the charge collection efficiency for a 20 $\mu$m pixel pad. Note that only a subset of non overlapping points is shown, which means that the point density is not representative of the actual distribution.

Figure 8.10: Detection efficiency as a function of pad size for various charge thresholds

Figure 8.11: Crosstalk signal fraction (left) and crosstalk rate (right) for various charge thresholds (right) of a neighbouring pixel. Error bars indicate the central 68.3% confidence region of the underlying distributions in the left plot, and uncertainties in the right plot. The crosstalk rate should be multiplied by four to get the total from the four nearest neighbours.
Figure 8.12: Signal induced on a neighbouring pixel pad of a Timepix3 chip by the same avalanche as in figure 8.5. At $t = 0$ ns, the initial electron crosses the grid hole. The top and bottom graphs show the integrated signal $Q_s$ and the current signal $I_s \equiv dQ_s/dt$. The left and right graphs have time scales that are appropriate for showing the electron and ion contributions to the signal.
9 Discussion and conclusion

9.1 Discussion

In this research we modelled a GridPix as described in section 2. We calculated the electric fields in section 6.2. We explained that we can treat the protection layer as a conductor for calculating the electric field. Its potential will be the same as that of the pixel pads. By using mirror boundary conditions, we found the electric field for pixels in the central region of the detector. Using these fields, we simulated electron avalanches in Garfield++.

We used the Shockley-Ramo theorem to calculate the signals that these avalanches induce. In section 3.1 we showed that the integrated signal only depends on the electron endpoints because the weighting potential is zero at the ion endpoints. In section 6.3 we also showed that ions that drift up into the drift region do not induce a significant signal any more after they pass the grid. Since the ions reach or pass the grid before the preamplifier output returns to baseline, we can ignore the time dependence of the ion signal—it only plays a role in timewalk.

Furthermore, in section 5.2 we explained that charge that sits on top of the protection layer dissipates on a time scale much longer than the time it takes for the preamplifier to return to baseline. Therefore, we treat the protection layer as an insulator when calculating the weighting field for a pixel pad.

Gain distribution

A Polya distribution is commonly used to describe the gain distribution. We showed that the gains as simulated by Garfield++ are also reasonably well described by a Polya distribution.

In [7] it is shown that the mean gain increases for electrons crossing the grid hole at a larger radius by integrating the Townsend coefficient along field lines. We also observed this effect by obtaining the gain using microscopic methods [25]. At larger radiiuses, integrating the Townsend coefficient significantly over-estimates the radial dependence compared to the microscopic method.

Charge collection efficiency

In figure 8.8 we saw that the charge collection efficiency first increases linearly as we increase the pixel pad size and then plateaus. We can understand qualitatively why this happens by comparing the distribution of electron endpoints in figure 8.4 to the weighting potential on the protection layer in figure 6.7. As we increase the pixel pad size, the weighting potential increases. However, the increase above the pixel pad stagnates at some point—The only way to increase it further is by making a thinner protection layer. The weighting potential still increases at larger radiiuses, but fewer electrons arrive there. As a result, the charge collection efficiency increases less for larger pixel pads.

On the other hand, the variation in charge collection efficiency from one event to another keeps decreasing. Figure 8.9 showed that, for a given pixel pad, the charge collection efficiency strongly depends on the position and size of the electron endpoint distribution. If an avalanche arrives further from the pixel centre, then a larger part of the electrons will see a low weighting potential. This is also true if the avalanche is spread out more over the pixel plane. We already noticed that the weighting potential keeps increasing at larger radiiuses for larger pixel pads. As a result, the variation in size and position of avalanches will affect the charge collection efficiency less if the pixel pad is bigger.
We found that the radial distribution of electrons on the protection layer is well described by the radial distribution of a bivariate Gaussian that is rotationally symmetric (equation 6.2). We also found that the mean charge collection efficiency can be calculated using this distribution according to equation 6.3. Therefore, predicting the variance of this distribution might be a more efficient way of calculating the charge collection efficiency for other field strengths, gases, and geometries (other grid height, protection layer thickness, grid hole size, etc.).

In our simulations, we have ignored the support pillars which we mentioned in section 1.3. Including them would not change the electric field. No bound charge will accumulate on the boundary surface between the pillars and the gas, since this surface is parallel to the electric field. Furthermore, the component of the electric field that is parallel to the boundary surface should be continuous. Therefore, the electric field must be the same inside the pillars.

Electrons that hit the pillars cannot cause any more ionisations, so including the pillars would decrease the gain. However, for 30 µm diameter pillars that are positioned between four pixels as shown in figure 1.3, only 0.09% of all electrons in our simulations would end up inside a pillar. If we would consider them as completely lost in calculating the induced signal, then it would decrease the mean charge collection efficiency by 0.06% for a 50 µm diameter pixel pad and less for smaller pads.

Variations in grid height have a larger impact on the charge collection efficiency. As mentioned in [7], the grid height varies by about 3 µm over a single wafer. Since the grid height varies smoothly, we expect the grid height variation among different chips to dominate over the grid height variation within a single chip. From the discussion in section 6.4, we know how to estimate the uncertainty in the charge collection efficiency (figure 8.8). For $\sigma(d_{\text{amp}}) \approx 3$ µm we find that $\sigma(\eta_{cc}) = 1.4\%$ for Timepix3 without enlarged pads and $\sigma(\eta_{cc}) = 0.22\%$ for a 50 µm diameter pixel pad.

Detection efficiency

We also explained that a voltage threshold at the discriminator corresponds to a certain charge threshold because Timepix3 has a charge sensitive preamplifier. This means that we can calculate the detection efficiency by counting the number of events for which the collected charge is larger than some charge threshold.

Our results in figure 8.10 showed that increasing the pixel pad size also increases the detection efficiency. The gain distribution and thickness of the protection layer determine the maximum possible detection efficiency for a given charge threshold. As we increase the pixel pad size, the distribution of collected charge will more closely resemble the gain distribution. As a result, the detection efficiency becomes very close to its maximum value.

Crosstalk

In figure 6.7 we saw that increasing the pixel pad size increases the weighting field in neighbouring pixel cells. This means that we will increase the crosstalk by enlarging the pixel pads. To measure this we calculated the charge collection and detection efficiencies in a neighbouring pixel cell by a translation of the weighting field. Our results in figure 8.12 show that crosstalk should not be an issue—even for 50 µm diameter pads.

9.2 Limitations

Most of the signal induced by an avalanche is due to ions. This means that the time dependent signal heavily depends on the ion mobility. However, we know relatively little about its value.
In section 2.3 we made a rough estimate by using Blanc’s law to combine the ion mobilities of CO$_2^+$ in CO$_2$ and DME$^+$ in DME. However, in reality the ion mobility can be different for CO$_2^+$ and DME$^+$ in the mixture CO$_2$:DME (50:50). Furthermore, a CO$_2^+$ might transfer its charge to a DME molecule or vice versa.

Despite these complications, we have been careful to make sure that the ion mobility does not affect our results (with the exception of the time dependent results in figures 8.5, 8.6, and 8.12). The shape of the ion signal does not affect our results for the charge collection efficiency, detection efficiency, and crosstalk—we only require that ions reach the grid fast enough as discussed above.

9.3 Suggestions for further research

We have only looked at avalanches in pixels in the central region of the detector. However, Gmsh and Elmer can also be used to calculate the electric fields between adjacent pixel chips in a study to minimise or understand how field deformations affect drifting electrons.

Another interesting study would be to use time dependent signals in combination with a complete analog front-end simulation. This can be used to examine what impact the relatively slow ion signal has on timewalk. Another subject that can be looked at is how the discriminator threshold corresponds to a charge threshold because the slow ion signal will result in a smaller amplitude of the preamplifier output. This will increase the minimum charge threshold that can be achieved, which plays a role in the detection efficiency for GridPix on Timepix3.

9.4 Conclusion

The results of our simulations showed that enlarging the pixel pads can significantly increase the charge collection efficiency by almost a factor two (figure 8.8). By enlarging the pixel pad to 50 µm in diameter, we can increase the mean charge collection efficiency from 54.0(14) % to 96.63(22) %, and decrease the standard deviation in charge collection efficiency among avalanches from 7 % to 0.7 %.

The increase in charge collection efficiency translates into an increase of the detection efficiency (figure 8.10). For a charge threshold of 500 e$^-$ the detection efficiency increases from 94.42(28) % to 97.60(28) %. For higher thresholds the increase is larger: for a charge threshold of 1000 e$^-$ the detection efficiency increases from 86.58(27) % to 93.84(28) %.

We also conclude that crosstalk should not become a problem. Even for a 50 µm diameter pad we found that only 0.43(4) % of the avalanches induced a hit in a neighbouring pixel.
A Analog front-end model

This section describes our model of the analog front-end. The goal of modelling the front-end is to have a complete GridPix simulation: (1) meshing the gas volume with Gmsh, (2) calculating the electric fields in Elmer, (3) simulating tracks in Heed, (4) charge transport in Garfield++, (5) calculating induced signals with weighting fields, (6) signal processing using the model described here, and (7) determining the ToA and ToT of each event. Such a simulation would be useful for studying timewalk corrections for example.

During the development of Timepix3, the analog front-end has already been properly simulated. The purpose here is to develop an approximate model that is fast enough to apply to a large number of events, but also good enough to study certain things such as time resolution and timewalk corrections. In its present condition, however, the model is incomplete (see section A.4).

Section A.1 starts with a description of the preamplifier. The remainder of this section develops a model of the preamplifier and mostly consists of algebra. This is also true for sections A.2, and A.3. The preamplifier models with and without noise are shown in algorithm A.1 on page 52 and in algorithm A.2 on page 56. Lastly, algorithm A.3 on page 58 shows a model of the discriminator. In section A.4 we discuss some of the shortcoming of our preamplifier model.

A.1 Preamplifier

Figure A.1 shows a charge-sensitive amplifier (CSA) with Krummenacher feedback [11]. The PMOS transistors $M_{1A}$ and $M_{1B}$ are identical and form a differential pair. The sum of their source-drain currents is equal to the current supplied on top: $I_{1A} + I_{1B} = I_{K}$. The ratio of these currents is mainly determined by the gate potentials $V_{fbk}$ and $V_{out}$. The current going into the capacitor $C_{l}$ is $I_{1B} - \frac{1}{2}I_{K}$ (remember that, barring parasitic capacitances, current cannot flow into the gate of a MOS transistor). The charge on $C_{l}$ controls the leakage current compensation $I_{2}$, because it determines the gate-source potential of the NMOS transistor $M_{2}$. Lastly, the sum of currents at the input node of the base amplifier, $I_{in} + I_{1A} - I_{2}$, is equal to the current going into the feedback capacitor $C_{f}$ (there is no current going into the base amplifier because internally its input is the gate of another NMOS transistor).

![Charge sensitive amplifier with Krummenacher feedback](image)

Figure A.1: Charge sensitive amplifier with Krummenacher feedback

To understand how this amplifier works, first consider the case in which there is no input
current \((I_{in} \text{ is zero})\). When the circuit is in equilibrium, the charge on both \(C_f\) and \(C_l\) must be constant, and therefore \(I_{1A} = I_2\) and \(I_{1B} = \frac{1}{2}I_k\). From before, we also know that \(I_{1A} + I_{1B} = I_k\), and thus \(I_{1A} = I_{1B}\). This means that, since \(M_{1A}\) and \(M_{1B}\) are identical and share the same source potential, their gate potentials must also be the same: \(V_{out} = V_{fbk}\). In other words, \(V_{fbk}\) sets the baseline of the preamplifier output.

We will now look at the feedback path that regulates the leakage current compensation. Suppose that a constant positive leakage current starts flowing into the preamplifier \((I_{in} = I_{leak})\). This current is integrated onto the feedback capacitor \(C_f\). This increases the input potential \(V_{in}\) and the base amplifier reacts by pulling down \(V_{out}\). As a result, \(I_{1B}\) increases and this additional current is integrated onto \(C_l\). This increases the current \(I_2\) and eventually the circuit reaches an equilibrium in which \(M_2\) sinks all of the leakage current: \(I_2 = \frac{1}{2}I_k + I_{leak}\).

To understand how the preamplifier reacts to transient signals, we will examine what happens when the input current is a short positive current pulse: \(I_{in} = Q\delta(t)\). Again, this current is integrated onto \(C_f\) which results in the base amplifier pulling down \(V_{out}\). The increase in \(I_{1B}\) is accompanied by a decrease in \(I_{1A}\). However, on this short time scale \(I_2\) does not change and the decrease in \(I_{1A}\) is compensated by \(C_f\) discharging through \(M_2\). For sufficiently large signals, the differential pair becomes completely unbalanced and \(C_f\) will be discharged by a constant current of \(\frac{1}{2}I_k\).

**Differential pair**

To model this preamplifier, we will first examine the differential pair formed by \(M_{1A}\) and \(M_{1B}\). We have isolated this part of the circuit in figure A.2. Here \(V_{g2}\) is the gate potential of \(M_2\) and \(V_{s1}\) is the source potential of both \(M_{1A}\) and \(M_{1B}\). The goal is to find an expression for \(I_{1A}\) in terms of \(V_{out}\).

![Figure A.2: The differential pair separated from the rest of the preamplifier in figure A.1](image)

Both \(M_{1A}\) and \(M_{1B}\) are operated in saturation. The source-drain current \(I_{sd}\) of a PMOS transistor in saturation is given by

\[
I_{sd} = \frac{1}{2}\mu_p C_{ox} \frac{W}{L} (V_{gs} - V_{th})^2
\]

(A.1)

Here \(\mu_p\) is the positive charge carrier mobility, \(C_{ox}\) is the oxide capacitance per unit area, \(W\) and \(L\) are the channel width and length, \(V_{gs}\) is the gate source potential difference \(V_g - V_s\), and \(V_{th}\) is the transistor’s threshold voltage. The values of \(\mu_p\), \(C_{ox}\), and \(V_{th}\) depend on the technology used. For the 130 nm IBM CMOS8RF process we have \(\mu_p C_{ox} \approx 90 \mu A V^{-2}\). In Timepix3 both \(M_{1A}\) and \(M_{1B}\) have \(W/L = 1/0.96\). We will write \(k_p \equiv \mu_p C_{ox} \frac{W}{L}\) and approximate it as \(k_p \approx 100 \mu A V^{-2}\).
Using equation A.1, we find that

\[ I_{1a} = \frac{k_p}{2} (V_{fbk} - V_{s1} - V_{th})^2 \]  

(A.2)

and,

\[ I_{1b} = \frac{k_p}{2} (V_{out} - V_{s1} - V_{th})^2 \]  

(A.3)

For conciseness, I will only sketch how we find our final expression for \( I_{1a} \). First we substitute equations A.2 and A.3 into \( I_{1a} + I_{1b} = I_k \) and solve for \( V_{s1} + V_{th} \). Then, we plug this expression for \( V_{s1} + V_{th} \) back into equation A.2. While doing all this, we need to bear in mind that a current can only flow through a PMOS transistor if the gate-source potential difference is more negative than the threshold voltage. Finally we find

\[
I_{1a}(V_{out}) = \begin{cases} 
\frac{1}{2} I_K + (V_{out} - V_{fbk}) \frac{k_p}{4} \sqrt{\frac{4 I_k}{k_p}} - (V_{out} - V_{fbk})^2 & \text{if } |V_{out} - V_{fbk}| < \sqrt{\frac{2 I_k}{k_p}} \\
0 & \text{if } V_{out} - V_{fbk} \leq -\sqrt{\frac{2 I_k}{k_p}} \\
I_K & \text{if } V_{out} - V_{fbk} \geq \sqrt{\frac{2 I_k}{k_p}} 
\end{cases}
\]  

(A.4)

Model

We will model the preamplifier around its equilibrium state. To do this, we redefine \( V_{out} \) and \( V_{in} \). From now on they are deviations from the equilibrium state. This also means that, for example, the charge \( Q_f \) calculated as \( (V_{in} - V_{out})C_f \) is also with respect to some equilibrium value that we do not actually know. Furthermore, we will make the approximation that the leakage current compensation does not change.

Figure A.3 shows how we will model the preamplifier. In this circuit we have also added the detector capacitance \( C_d \). We have written the discharge current as \( I_d = \frac{1}{2} I_K - I_{1a} \). Using equation A.4 we have

\[
I_d(V_{out}) = \begin{cases} 
-V_{out} \frac{k_p}{4} \sqrt{\frac{4 I_k}{k_p}} - V_{out}^2 & \text{if } |V_{out}| < \sqrt{\frac{2 I_k}{k_p}} \\
\frac{1}{2} I_K & \text{if } V_{out} \leq -\sqrt{\frac{2 I_k}{k_p}} \\
-\frac{1}{2} I_K & \text{if } V_{out} \geq \sqrt{\frac{2 I_k}{k_p}} 
\end{cases}
\]  

(A.5)

Figure A.4 shows a plot of this function. We will model the base amplifier as a single-pole transfer function:

\[ \tilde{V}_{out} = -\frac{A_0}{1 + s \tau_0} \tilde{V}_{in} \]  

(A.6)

Here \( A_0 \) and \( 1/\tau_0 \) are the open-loop values of its gain and bandwidth, and \( \tilde{\cdot} \) denotes the Laplace domain.

Before we continue, I will mention a property of the Laplace transform that we will need shortly. In calculating the signal of an avalanche, it is more straightforward to sample the integrated signal \( Q_s(t) \) than its derivative \( I_s(t) \). Therefore, we will use the integration property

\[
\int_0^t dt' f(t') \quad \xrightarrow{\mathcal{L}} \quad \frac{1}{s} \tilde{f}(s) 
\]  

(A.7)

to write \( \tilde{Q} \equiv \frac{1}{s} \tilde{I} \), where \( Q(t) \equiv \int_0^t dt' I(t') \).
We will now find the transfer function from $\tilde{Q}_s - \tilde{Q}_d$ to $\tilde{V}_{\text{out}}$ (we cannot derive a transfer function directly from $\tilde{Q}_s$ to $\tilde{V}_{\text{out}}$, because the discharge current $I_d$ is nonlinear in $V_{\text{out}}$). The potential difference across the feedback capacitor is

$$\tilde{V}_{\text{in}} - \tilde{V}_{\text{out}} = \tilde{I}_f Z_f$$  \hfill (A.8)

and the input node potential is

$$\tilde{V}_{\text{in}} = \tilde{I}_d Z_d$$  \hfill (A.9)

where $Z_f^{-1} \equiv s C_f$, and $Z_d^{-1} \equiv s C_d$. Furthermore, Kirchhoff’s current law tells us that

$$I_s - I_d - I_f - I_d = 0$$  \hfill (A.10)

Using equations A.9 and A.10, we rewrite the right-hand side of equation A.8 as

$$\tilde{I}_f Z_f = \left( \tilde{I}_s - \tilde{I}_d - \frac{\tilde{V}_{\text{in}}}{Z_d} \right) Z_f$$

Substituting this back into equation A.8 and rearranging gives us

$$\tilde{V}_{\text{in}} \left( 1 + \frac{C_d}{C_f} \right) - \tilde{V}_{\text{out}} = \left( \tilde{I}_s - \tilde{I}_d \right) Z_f$$

Next, we use equation A.6 to express $\tilde{V}_{\text{in}}$ in terms of $\tilde{V}_{\text{out}}$:

$$- \tilde{V}_{\text{out}} \left[ 1 + \frac{1 + s \tau_0}{A_0} \left( 1 + \frac{C_d}{C_f} \right) \right] = \left( \tilde{I}_s - \tilde{I}_d \right) Z_f$$  \hfill (A.11)
If $A_0$ is large enough, then we can approximate $(1 + s \tau_0)/A_0 \approx s \tau_0/A_0$. The term in square brackets becomes
\[
1 + \left(1 + \frac{C_D}{C_F} \right) \approx 1 + \frac{C_D}{C_F} \cdot s \tau_r
\]
where we have defined the rise-time parameter $\tau_r \equiv \tau_0(1 + C_D/C_F)/A_0$. Using the definition of $Z_f$ and the property in equation A.7, we cast equation A.11 into its final form:
\[
\tilde{V}_{\text{out}} = -\frac{1}{C_f} \frac{1}{1 + s \tau_r} \left( \tilde{Q}_s - \tilde{Q}_d \right)
\]
(A.12)

So, the transfer function is
\[
\tilde{h}(s) \equiv \frac{\tilde{V}_{\text{out}}}{Q_s - Q_d} = \frac{1}{C_f} \frac{1}{1 + s \tau_r} \quad \mathcal{L} \quad h(t) = -\frac{1}{C_r \tau_r} e^{-t/\tau_r}, \quad t \geq 0
\]
(A.13)

We will now switch to discrete-time signals that are sampled in steps of $\Delta t$, and denote them using square brackets: $f[n] \equiv f(n \Delta t)$. The first thing that we will do is to express the discrete convolution of the transfer function $h[n]$ and some other function $f[n]$ as a recurrence relation by using that $h(t)$ is an exponential, and therefore
\[
h(t + \Delta t) = e^{-\Delta t/\tau_r} h(t)
\]
(A.14)

The discrete convolution of $h[n]$ and $f[n]$ is
\[
(h * f)[n] = \sum_{m=0}^\infty h[m] f[n - m] \Delta t
\]
\[
= h[0] f[n] \Delta t + \sum_{m=1}^\infty h[m] f[n - m] \Delta t
\]
\[
= h[0] f[n] \Delta t + \sum_{m=0}^\infty h[m + 1] f[n - 1 - m] \Delta t
\]
(A.15)

Using equation A.14, we can rewrite the second term on the right-hand side as
\[
\sum_{m=0}^\infty h[m + 1] f[n - 1 - m] \Delta t = e^{-\Delta t/\tau_r} \sum_{m=0}^\infty h[m] f[n - 1 - m] \Delta t
\]
\[
e^{-\Delta t/\tau_r} (h * f)[n - 1]
\]

Plugging this back into equation A.15 gives us the recurrence relation for the discrete convolution of $h[n]$ and $f[n]$:
\[
(h * f)[n] = -\frac{\Delta t}{C_f \tau_r} f[n] + e^{-\Delta t/\tau_r} (h * f)[n - 1]
\]
(A.16)

Now we can explain how to calculate the preamplifier response of some input signal $Q_s[n]$. We require that the signal arrives after $t = 0$ and that $Q_s[0] = 0$. We also set the initial conditions $V_{\text{out}}[0] = 0$ and $Q_d[0] = 0$. Using equation A.16, we know that
\[
V_{\text{out}}[n] = (h * (Q_s - Q_d))[n]
\]
\[
= -\frac{\Delta t}{C_f \tau_r} (Q_s[n] - Q_d[n]) + e^{-\Delta t/\tau_r} V_{\text{out}}[n - 1]
\]
(A.17)

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The value of $Q_s[n]$ is given, but we need to find $Q_d[n]$ ourselves. For our purposes, a sufficient approximation is

$$Q_d[n] = Q_{d}[n - 1] + I_d(V_{\text{out}}[n - 1]) \Delta t$$ (A.18)

Here we calculate the discharge current $I_d$ as a function of $V_{\text{out}}$ using equation A.5. So, we first calculate $Q_d[1]$ with equation A.18, and then we can calculate $V_{\text{out}}[1]$ with equation A.17. Next we calculate $Q_d[2]$ and $V_{\text{out}}[2]$, $Q_d[3]$ and $V_{\text{out}}[3]$, and so forth. Algorithm A.1 shows the crux of this procedure and table A.1 contains the values that we use for the various parameters.

**Algorithm A.1** Calculating the preamplifier response. See table A.1 for more information on the parameters.

**Input:** The integrated signal $Q_s[n]$ for $0 < n < N$. ($Q_s[0]$ is assumed to be zero.)

**Output:** The preamplifier response $V_{\text{out}}[n]$ for $0 \leq n < N$

1: $Q_d \leftarrow 0$
2: $V_{\text{out}}[0] \leftarrow 0$
3: for $n \leftarrow 1, N - 1$
   4: $Q_d \leftarrow Q_d + I_d(V_{\text{out}}[n - 1]) \Delta t$ \hspace{1cm} $\triangleright I_d(V_{\text{out}})$ from equation A.5
   5: $V_{\text{out}}[n] \leftarrow -\frac{\Delta t}{C_f \tau_r} (Q_s[n] - Q_d) + e^{-\Delta t/\tau_r} V_{\text{out}}[n - 1]$
4: end for

**Table A.1:** Parameters used in in the preamplifier model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_f$</td>
<td>3 fF</td>
<td>Feedback capacitor</td>
</tr>
<tr>
<td>$k_p$</td>
<td>100 $\mu$A V$^{-2}$</td>
<td>Gain factor of M$<em>{1A}$ and M$</em>{1B}$ (page 48)</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>2.8 ns</td>
<td>Rise-time parameter (page 51)</td>
</tr>
<tr>
<td>$I_k$</td>
<td>1.2 nA</td>
<td>Krummenacher current</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>0.1 ns</td>
<td>Simulation time-step</td>
</tr>
</tbody>
</table>

### A.2 Noise

In this section, we will add noise to the preamplifier output of our model. This noise should have zero mean and the correct variance:

$$\text{var}(V_{\text{out}}[n]) = \left(\frac{Q_{\text{ENC}}}{C_f}\right)^2$$ (A.19)

where $Q_{\text{ENC}}$ is the equivalent noise-charge (ENC). A common method to model amplifier noise is to add two noise sources at the input: a current source in parallel and a voltage source in series. Simply put, the idea behind this is that noise on the output is often dominated by noise from a component somewhere at the input. In figure A.5 we added two noise sources to our model.

To redo our circuit analysis, we need to replace $\tilde{V}_{\text{in}}$ in equation A.6 with $\tilde{V}_{\text{in}} + \tilde{V}_n$ and add $I_n$ to the left-hand side of equation A.10. This changes equation A.12 to

$$\tilde{V}_{\text{out}} = -\frac{1}{C_f} \frac{1}{1 + s \tau_r} \left( \dot{Q}_s + \frac{1}{s} I_n + (C_f + C_d) \tilde{V}_n - \dot{Q}_d \right)$$ (A.20)
We can combine the two noise sources into a single term:

$$\tilde{Q}_n \equiv \frac{1}{s} \tilde{I}_n + (C_f + C_d) \tilde{V}_n$$

Equation A.17 now becomes

$$V_{\text{out}}[n] = -\frac{\Delta t}{C_f \tau_f} \left( Q_s[n] + Q_n[n] - Q_d[n] \right) + e^{-\Delta t/n} V_{\text{out}}[n-1] \quad (A.21)$$

In Timepix3, the noise is dominated by the voltage noise, so we will set $I_n = 0$. Furthermore, we will model $V_n(t)$ as a Gaussian white-noise process. This means that $Q_n(t)$ is also a Gaussian white-noise process. In our discrete-time model, the samples $Q_n[n]$ are independent and Gaussian distributed.

The remainder of this section describes how to find the correct variance for generating the Gaussian distributed noise $Q_n[n]$ and how to generate initial values for $V_{\text{out}}[0]$ and $Q_d[0]$ to mimic an infinite history of noise before $t = 0$. This is a rather monotonous affair and the model with noise can be found on page 56.

Now we will determine what variance in $Q_n[n]$ results in the correct variance in $V_{\text{out}}[n]$. We could do this by trial and error, but it is easier to perform a small-signal analysis. To do this, we make a second-order approximation of the discharge current $I_d(V_{\text{out}})$ around $V_{\text{out}} = 0$. Using equation A.5, we have

$$I_d(V_{\text{out}}) = -\frac{1}{2} \sqrt{f_k k_p} V_{\text{out}} + O(V_{\text{out}}^3) \approx -\frac{1}{R} V_{\text{out}} \quad (A.22)$$

with $R \equiv 2/\sqrt{f_k k_p}$. With this, equation A.18 becomes

$$Q_d[n] = Q_d[n-1] - \frac{\Delta t}{R} V_{\text{out}}[n-1] \quad (A.23)$$

To calculate $V_{\text{out}}[n]$ directly from $Q_n[n]$, we need to combine equations A.21 and A.23 to avoid having to calculate $Q_d[n]$. For this we will use the $Z$ transform:

$$f[n] \xrightarrow{Z} \tilde{f}(z) \equiv \sum_{n=0}^{\infty} f[n] z^{-n}$$

where $\tilde{}$ denotes the $Z$ domain. Since equations A.21 and A.23 are recurrence relations, a useful property of this transform is that

$$f[n-k] \xrightarrow{Z} \frac{1}{z^k} \tilde{f}(z)$$
In the \( Z \) domain, equations A.21 and A.23 read
\[
V_{\text{out}} = -\frac{\Delta t}{C_f \tau_f} \left( \bar{Q}_s + \bar{Q}_n - \bar{Q}_d \right) + e^{-\Delta t/\tau_f} \frac{1}{z} V_{\text{out}} \quad (\text{A.24})
\]
and
\[
\bar{Q}_d = \frac{1}{z} \left( \bar{Q}_d - \frac{\Delta t}{R} V_{\text{out}} \right) \quad (\text{A.25})
\]
Solving these equations to cancel \( \bar{Q}_d \) gives us
\[
V_{\text{out}} = -\frac{\Delta t}{C_f \tau_f} \left( \frac{(z - 1) z}{(z - a_+)(z - a_-)} \right) \left( \bar{Q}_s + \bar{Q}_n \right)
\]
with
\[
a_{\pm} = \frac{1}{2} \left[ \left( 1 + e^{-\Delta t/\tau_f} - \frac{\Delta t^2}{RC_f \tau_f} \right) \pm \sqrt{\left( 1 + e^{-\Delta t/\tau_f} - \frac{\Delta t^2}{RC_f \tau_f} \right)^2 - 4 e^{-\Delta t/\tau_f}} \right]
\]
This gives us the transfer function
\[
g_1(z) \equiv -\frac{\Delta t}{C_f \tau_f} \frac{(z - 1) z}{(z - a_+)(z - a_-)} \quad \leftrightarrow \quad g_1[n] = -\frac{\Delta t}{C_f \tau_f} \frac{(a_+ - 1) a_n^n - (a_- - 1) a_n^n}{a_+ - a_-}, \; n \geq 0
\]
Without signal, the preamplifier output is
\[
V_{\text{out}}[n] = \sum_{k=0}^{\infty} g[k] Q_n[n - k]
\]
Now we can calculate the variance of \( V_{\text{out}}[n] \):
\[
\text{var}(V_{\text{out}}[n]) = E \{ V_{\text{out}}[n]^2 \} = \sum_{k=0}^{\infty} g_1[k] Q_n[n - k] \sum_{k'=0}^{\infty} g_1[k'] Q_n[n - k']
\]
\[
= \sum_{k=0}^{\infty} \sum_{k'=0}^{\infty} g_1[k] g_1[k'] E \{ Q_n[n - k] Q_n[n - k'] \} = \sum_{k=0}^{\infty} \sum_{k'=0}^{\infty} g_1[k] g_1[k'] \text{cov}(Q_n[n - k], Q_n[n - k'])
\]
Since the samples \( Q_n[n] \) are independent, we have
\[
\text{cov}(Q_n[n_1], Q_n[n_2]) = \begin{cases} \text{var}(Q_n[n]) & \text{if } n_1 = n_2 \\ 0 & \text{if } n_1 \neq n_2 \end{cases}
\]
So, we find that
\[
\text{var}(V_{\text{out}}[n]) = \text{var}(Q_n[n]) \sum_{k=0}^{\infty} g_1[k]^2
\]
which gives us
\[
\text{var}(Q_n[n]) = \text{var}(V_{\text{out}}[n]) \left( \sum_{k=0}^{\infty} g_1[k]^2 \right)^{-1}
\]
\[
= \text{var}(V_{\text{out}}[n]) \frac{1}{2} \left( \frac{C_f \tau_f}{\Delta t} \right)^2 (1 + a_+) (1 + a_-) (1 - a_+ a_-)
\]
\[
= \text{var}(V_{\text{out}}[n]) C_f^2 \left( 1 - e^{-\Delta t/\tau_f} \right) \left[ \left( 1 + e^{-\Delta t/\tau_f} \right) \left( \frac{\tau_f}{\Delta t} \right)^2 - \frac{\tau_f}{2 R C_f} \right] \quad (\text{A.26})
\]

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Plugging in equation A.19, we find the variance of $Q_n[n]$:

$$\text{var}(Q_n[n]) = Q_{\text{ENC}}^2 \left( 1 - e^{-\Delta t/\tau_f} \right) \left[ (1 + e^{-\Delta t/\tau_f}) \left( \frac{\tau_f}{\Delta t} \right)^2 - \frac{\tau_f}{2 R C_f} \right]$$  \hspace{1cm} (A.27)

Next, we need to determine the initial values $V_{out}[0]$ and $Q_d[0]$. The value of $V_{out}[0]$ is just a Gaussian random number with variance $(Q_{\text{ENC}}/C_f)^2$. The value of $Q_d[0]$ is also a Gaussian random number, but with a different variance. Furthermore, $Q_d[0]$ is correlated with $V_{out}[0]$.

We can use equations A.24 and A.25 to cancel $V_{out}$ and find

$$\tilde{Q}_d = \frac{\Delta t^2}{RC_f \tau_f} \frac{z}{(z - a_+)(z - a_-)} \left( \bar{Q}_s + \bar{Q}_n \right)$$

which gives us the transfer function

$$\bar{g}_2(z) \equiv \frac{\Delta t^2}{RC_f \tau_f} \frac{z}{(z - a_+)(z - a_-)} \quad \leftrightarrow \quad g_2[n] = \frac{\Delta t^2}{RC_f \tau_f} \frac{a_+^n - a_-^n}{a_+ - a_-}, \quad n \geq 0$$

We find the variance of $Q_d[n]$ the same way as before:

$$\text{var}(Q_d[n]) = \text{var}(Q_n[n]) \sum_{k=0}^{\infty} g_2[k]^2$$

$$= \text{var}(Q_n[n]) \left( \frac{\Delta t^2}{RC_f \tau_f} \right)^2 \frac{(1 + a_+ a_-)}{(1 - a_+^2)(1 - a_-^2)}$$

Plugging in equation A.26 gives

$$\text{var}(Q_d[n]) = \text{var}(V_{out}[n]) \frac{1}{2} \left( \frac{\Delta t}{R} \right)^2 \frac{(1 + a_+ a_-)}{(1 - a_+)(1 - a_-)}$$

$$= \text{var}(V_{out}[n]) \frac{C_f \tau_f}{2 R} \left( 1 + e^{-\Delta t/\tau_f} \right)$$

and thus

$$\text{var}(Q_d[n]) = Q_{\text{ENC}}^2 \frac{\tau_f}{2 R C_f} \left( 1 + e^{-\Delta t/\tau_f} \right)$$  \hspace{1cm} (A.28)

To calculate $\text{cov}(V_{out}[n], Q_d[n])$, we calculate $\text{var}(Q_d[n])$ using equation A.23:

$$\text{var}(Q_d[n]) = \text{var}(Q_d[n]) + \left( \frac{\Delta t}{R} \right)^2 \text{var}(V_{out}[n]) - \frac{2 \Delta t}{R} \text{cov}(V_{out}[n], Q_d[n])$$

By subtracting $\text{var}(Q_d[n])$ and plugging in equation A.19, we find that

$$\text{cov}(V_{out}[n], Q_d[n]) = \frac{\Delta t}{2 R} \left( \frac{Q_{\text{ENC}}}{C_f} \right)^2$$

Finally, the correlation between $V_{out}[n]$, and $Q_d[n]$ is

$$\text{corr}(V_{out}[n], Q_d[n]) = \frac{\text{cov}(V_{out}[n], Q_d[n])}{\sqrt{\text{var}(V_{out}[n]) \text{var}(Q_d[n])}}$$

$$= \frac{\Delta t}{\sqrt{2 RC_f \tau_f} (1 + e^{-\Delta t/\tau_f})}$$  \hspace{1cm} (A.29)

Algorithm A.2 shows the preamplifier model with noise. Table A.2 contains the additional parameters for this model.
Algorithm A.2 Calculating the preamplifier response with noise. See tables A.1, and A.2 for more information on the parameters.

Input: The integrated signal $Q_s[n]$ for $0 < n < N$. ($Q_s[0]$ is assumed to be zero.)
Output: The preamplifier response $V_{out}[n]$ for $0 \leq n < N$

1: $r_1 \leftarrow$ RandomGaussian()
2: $r_2 \leftarrow$ RandomGaussian()
3: $V_{out}[0] \leftarrow r_1 V_{out}^{rms}$
4: $Q_d \leftarrow (r_1 \rho + r_2 \sqrt{1 - \rho^2}) Q_{d}^{rms}$
5: for $k \leftarrow 1, N - 1$ do
6: $Q_n \leftarrow$ RandomGaussian() $Q_{n}^{rms}$
7: $Q_d \leftarrow Q_d + I_d(V_{out}[k-1]) \Delta t$ \hspace{1cm} $\triangleright$ $I_d(V_{out})$ from equation A.5
8: $V_{out}[k] \leftarrow -\frac{\Delta t}{C_f \tau} (Q_s[k] + Q_n - Q_d) + e^{-\Delta t/\tau} V_{out}[k-1]$
9: end for

Table A.2: Parameters used in the preamplifier model with noise in addition to the ones from table A.1. Note that we are only free to choose $Q_{enc}$—the others are derived parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculated as</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{enc}$</td>
<td>$75 \ e^-$</td>
<td>Equivalent noise charge</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$2/\sqrt{R K_p}$</td>
<td>$5.8 \ \text{M} \Omega$</td>
<td>Eq. A.22</td>
</tr>
<tr>
<td>$\rho$</td>
<td>corr($V_{out}[n], Q_d[n]$)</td>
<td>$7.2 \times 10^{-3}$</td>
<td>Eq. A.29</td>
</tr>
<tr>
<td>$V_{out}^{rms}$</td>
<td>$\sqrt{\text{var}(V_{out}[n])}$</td>
<td>$4.0 \ \text{mV}$</td>
<td>Eq. A.19</td>
</tr>
<tr>
<td>$Q_{d}^{rms}$</td>
<td>$\sqrt{\text{var}(Q_d[n])}$</td>
<td>$30 \ e^+$</td>
<td>Eq. A.28</td>
</tr>
<tr>
<td>$Q_{n}^{rms}$</td>
<td>$\sqrt{\text{var}(Q_n[n])}$</td>
<td>$551 \ e^+$</td>
<td>Eq. A.27</td>
</tr>
</tbody>
</table>
A.3 Discriminator

Here we will discuss a simple discriminator model (figure A.6) that is based on the discussion of discriminators in [13]. First, the discriminator amplification stage (das) amplifies the difference between the preamplifier output \( V_{\text{out}} \) and the threshold setting \( V_{\text{thr}} \). The CMOS logic-inverter converts the das-output \( V_{\text{das}} \) into a well-defined logic signal \( V_{\text{dis}} \) and also serves as a buffer to drive an output load.

![A conceptual discriminator](image)

**Figure A.6:** A conceptual discriminator

We will assume that the amplification stage can be modelled with a single-pole transfer function:

\[
\ddot{V}_{\text{das}} = \frac{A_{\text{das}}}{1 + s \tau_{\text{das}}} (\dot{V}_{\text{out}} - \dot{V}_{\text{thr}})
\]

The gain \( A_{\text{das}} \) can be expressed in the minimum potential difference between the inputs that makes the output swing from ground to \( V_{dd} \). We will use the preamplifier gain to write this potential difference in terms of a minimum charge:

\[
A_{\text{das}} = \frac{V_{dd} \Delta t}{Q_{\text{min}}} = \frac{V_{dd} C_{f} \Delta t}{Q_{\text{min}}}
\]

In the same way as before, we construct the discrete model:

\[
V_{\text{das}}[n] = \frac{V_{dd} C_{f} \Delta t}{Q_{\text{min}} \tau_{\text{das}}} (V_{\text{out}}[n] - V_{\text{thr}}) + e^{-\Delta t/\tau_{\text{das}}} V_{\text{das}}[n-1]
\]

After each time step we will constrain \( |V_{\text{das}}[n]| \) to be less than \( \frac{1}{2}V_{dd} \). The discriminator output \( V_{\text{dis}}[n] \) is \( +\frac{1}{2}V_{dd} \) if \( V_{\text{das}}[n] > 0 \) and vice versa.\(^3\) To calculate the initial value \( V_{\text{das}}[0] \) we assume that the preamplifier output has been sitting at \( V_{\text{out}}[0] \) long enough for \( V_{\text{das}} \) to stabilise. This might be a problem when the threshold is close to the noise. In that case it might be better to delay the original signal \( Q_{s} \) by at least a few multiples of \( \tau_{\text{das}} \).

Algorithm A.3 shows the discriminator model, but it is incomplete since we have no estimates for the parameters \( Q_{\text{min}} \) and \( \tau_{\text{das}} \). Figure A.7 shows how the discriminator responds to step signals of various sizes and illustrates that the discriminator output switches faster for larger signals (figure A.8). The signals start out well below threshold so that \( V_{\text{das}} \) is pulled to its lowest possible value.

\(^3\)We are ignoring any inverting of the signal in the discriminator stages.
Algorithm A.3 Calculating the discriminator response

**Input:** The preamplifier response $V_{\text{out}}[n]$ for $0 \leq n < N$

**Output:** The discriminator responses $V_{\text{das}}[n]$ and $V_{\text{dis}}[n]$ for $0 \leq n < N$

1: function $\text{Cap}(V)$
2: return $\text{Sign}(V) \cdot \text{Min}(|V|, \frac{1}{2}V_{\text{dd}})$
3: end function

4: $V_{\text{das}}[0] \leftarrow \text{Cap}\left(\frac{V_{\text{dd}}C_f}{Q_{\text{min}}} (V_{\text{out}}[0] - V_{\text{thr}})\right)$
5: $V_{\text{dis}}[0] \leftarrow \text{Sign}(V_{\text{das}}[0]) \cdot \frac{1}{2}V_{\text{dd}}$
6: for $k \leftarrow 1, N - 1$ do
7: $V_{\text{das}}[k] \leftarrow \text{Cap}\left(\frac{V_{\text{dd}}C_f \Delta t}{Q_{\text{min}} \tau_{\text{das}}} (V_{\text{out}}[k] - V_{\text{thr}}) + e^{-\Delta t/\tau_{\text{das}}} V_{\text{das}}[k - 1]\right)$
8: $V_{\text{dis}}[k] \leftarrow \text{Sign}(V_{\text{das}}[k]) \cdot \frac{1}{2}V_{\text{dd}}$
9: end for

Figure A.7: Discriminator output (top) and amplification stage output (middle) for step signals of various sizes (bottom). Note that we added $\frac{1}{2}V_{\text{dd}}$ to $V_{\text{dis}}$ and $V_{\text{das}}$, because our model calculates the discriminator outputs with respect to that value.

Figure A.8: Discriminator propagation-delay versus the over-threshold voltage of the step signal. At $t = 0$, the input signal steps from $\frac{1}{2}Q_{\text{min}}/C_f$ below threshold to $V_{\text{overdrive}}$ above threshold.
A.4 Improvements

Figure A.9 shows a comparison of our preamplifier model and a Cadence simulation. The Cadence simulation contains a number of features that our simple model does not reproduce.

Firstly, the constant discharge current is actually different for positive and negative signals. We might reproduce this by defining a new discharge current using equation A.5:

$$I_d'(V_{out}) = I_d(V_{out} + V_{offset}) - I_d(V_{offset})$$  (A.30)

where the parameter $V_{offset}$ controls the difference in the discharge current. Referring back to figure A.1, this basically means that the differential pair formed by $M_{1A}$ and $M_{1B}$ is slightly out of balance when the preamplifier is in equilibrium.

We might also want to change the value of $k_p$ to $1 \mu A V^{-2}$. Note that this is very different from our initial estimate of $100 \mu A V^{-2}$. Figure A.10 shows the effect of this change in combination with the new discharge current from equation A.30.

This analog front-end model still requires some work. It lacks the overshoot when the preamplifier returns to baseline. Also, for the $\pm 1 k e^-$ signals, the discharge current should build up more slowly after the signal arrives. Both of these shortcomings are most likely because our discharge current reacts instantly to changes in $V_{out}$. Furthermore, this model
also lacks the tiny overshoot that is a result of the leakage-current compensation. Lastly, the 10 ke\textsuperscript{−} signal response from the Cadence simulation in figure A.9 appears to have an amplitude of a 9 ke\textsuperscript{−} signal. Based on how the signal returns to baseline, this does not look like a saturation effect. At this point, we cannot explain this feature.
References


