The plasma properties of the QGP

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- Liquid
- Gas
- Solid

$\Big\}$ $< 5\%$ of 'light' (not-dark) matter in universe

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Contents

Section 1

Plasma Definition

Definition

A 'gas' of freely moving charged particles (that is overal neutral), where the process of screening prevents long-range particle interactions, but which interactions are still dominated by the forces associated with the charge.

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Charges have to be in unbound state.
Otherwise the screening will not occur
To be explained
Otherwise (extremely dilute plasma) it would be effectively the same as a neutral gas

Definition (2)

But often no hard phase transition. Plasma description and equations are usable beyond the strict definition of plasma.

- Free electrons in a metal
  Technically not completely free, so some people say it is a plasma, some it is plasma-like.
- Stellar systems
  Since gravity has only positive charges, screening doesn't excist. However gravity and the Coriolis force do result in equations that can be interpreted as a "gravitational plasma".

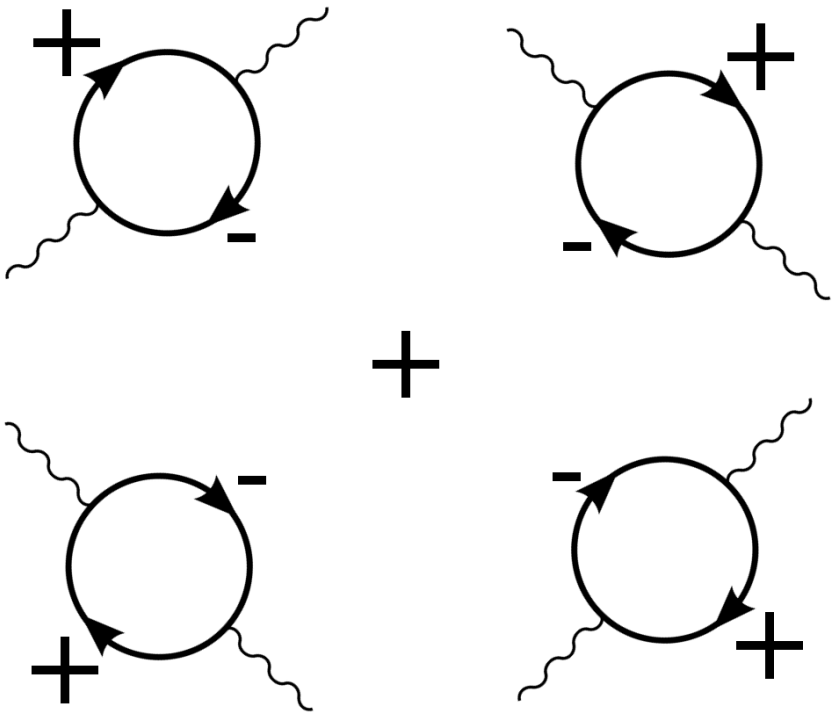
Section 2

Screening and Debye Length

Screening in QED: easy example

- Charge in vacuum
- Vacuum diagrams
- Polarization
- Influence potential: less force at a larger distance

$\Rightarrow$ Debye screening length r_D

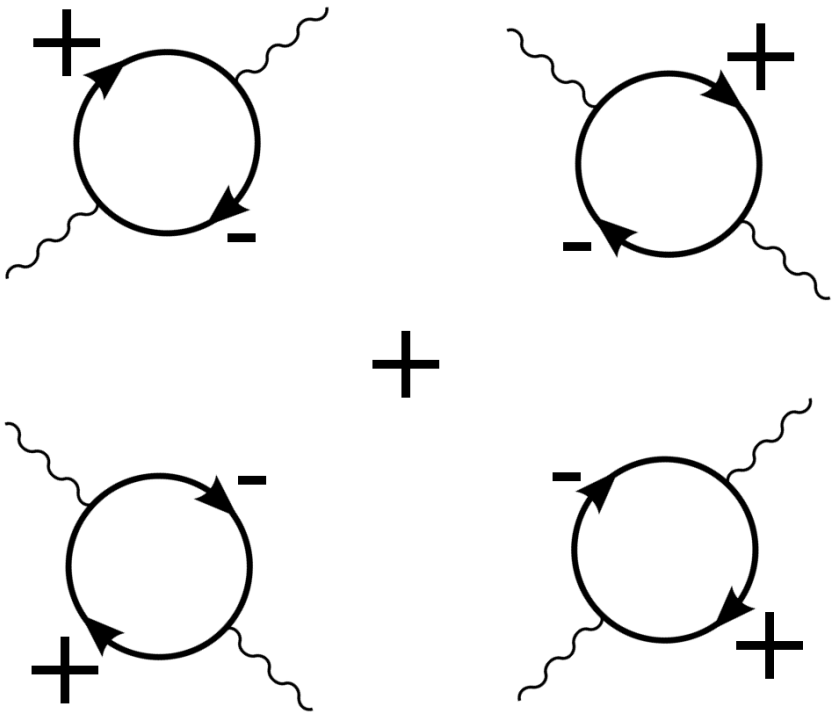


Screening in QED: easy example

$\Rightarrow$ Debye screening length r_D

$$r_D = \sqrt{\frac{\epsilon_0 kT}{\overline{n}e^2}}$$

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0 r} e^{-\sqrt{2}r/r_D}$$



Debye number

Debye number N_D or **Plasma parameter**:
Number of charged particles within a sphere of Debye radius.

- $N_D > 1$
- $\Lambda_{\mathrm{MFP,elec}} << r_D$

Typical Debye Length

Plasma	n_e (m^{-3})	T (K)	B (T)	λ_D (m)	N_D	ω_p (s^{-1})
Gas discharge	10^{16}	10^4	—	10^{-4}	10^4	10^{10}
Tokamak	10^{20}	10^8	10	10^{-4}	10^8	10^{12}
Ionosphere	10^{12}	10^3	10^{-5}	10^{-3}	10^5	10^8
Magnetosphere	10^7	10^7	10^{-8}	10^2	10^{10}	10^5
Solar core	10^{32}	10^7	—	10^{-11}	1	10^{18}
Solar wind	10^6	10^5	10^{-9}	10	10^{11}	10^5
Interstellar medium	10^5	10^4	10^{-10}	10	10^{10}	10^4
Intergalactic medium	1	10^6	—	10^5	10^{15}	10^2

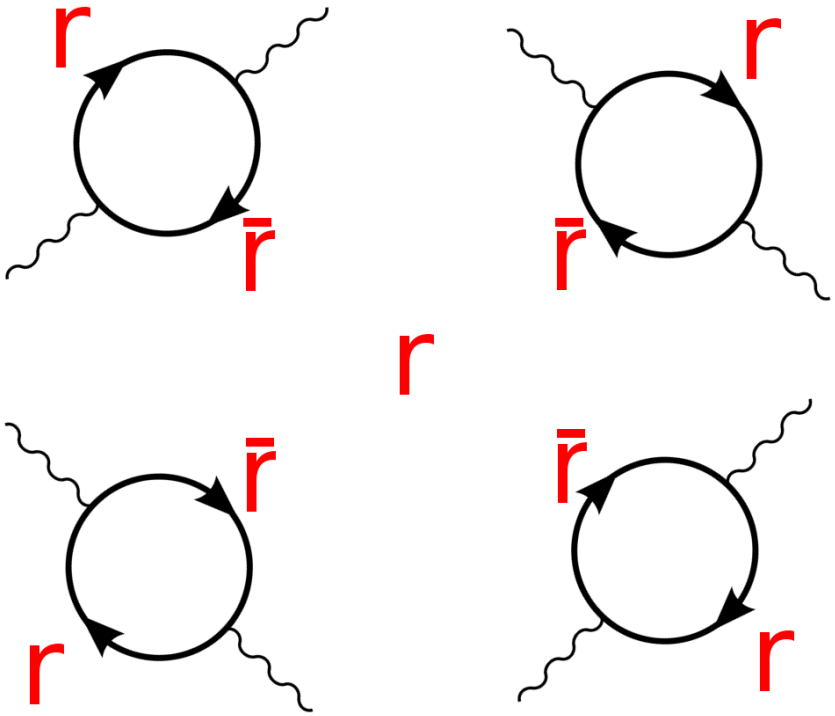
Blandford, Thorne: **Lecture notes for 'Applications of Classical Physics'**

`http://www.cns.gatech.edu/PHYS-4421/caltech136/` (2000)

Screening in QCD

Analogously with strong force instead of electro-magnetic.

Quark with color orientates quark-anti-quark pairs:



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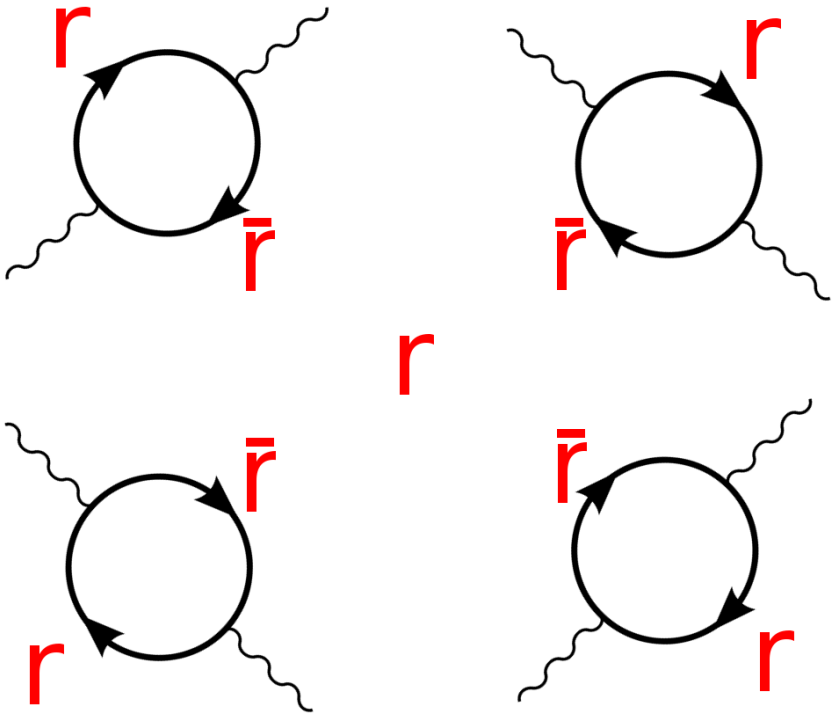


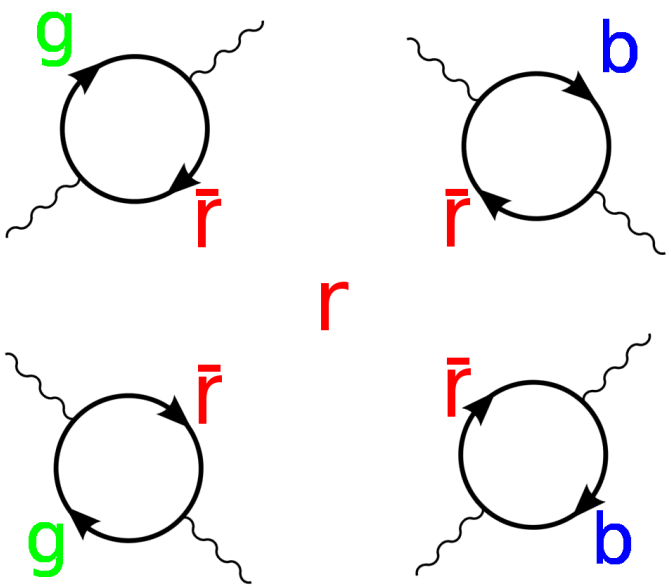
Anti-screening in QCD

Contrary to QED, QCD has 3-gluon and 4-gluon vertices:
Gluons have color

Polarization after gluon self-interactions

$\Downarrow$

Anti-screening: Changes the color



Anti-screening in QCD

Contrary to QED, QCD has 3-gluon and 4-gluon vertices:
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$$\Downarrow$$

Anti-screening: Changes the color

Knowledge on subject gained through Latice QCD.
Dictates a 'Debye Screening Length' $r_D(T)$, like in electromagnetism, only then dependent on temperature.

Lattice QCD results

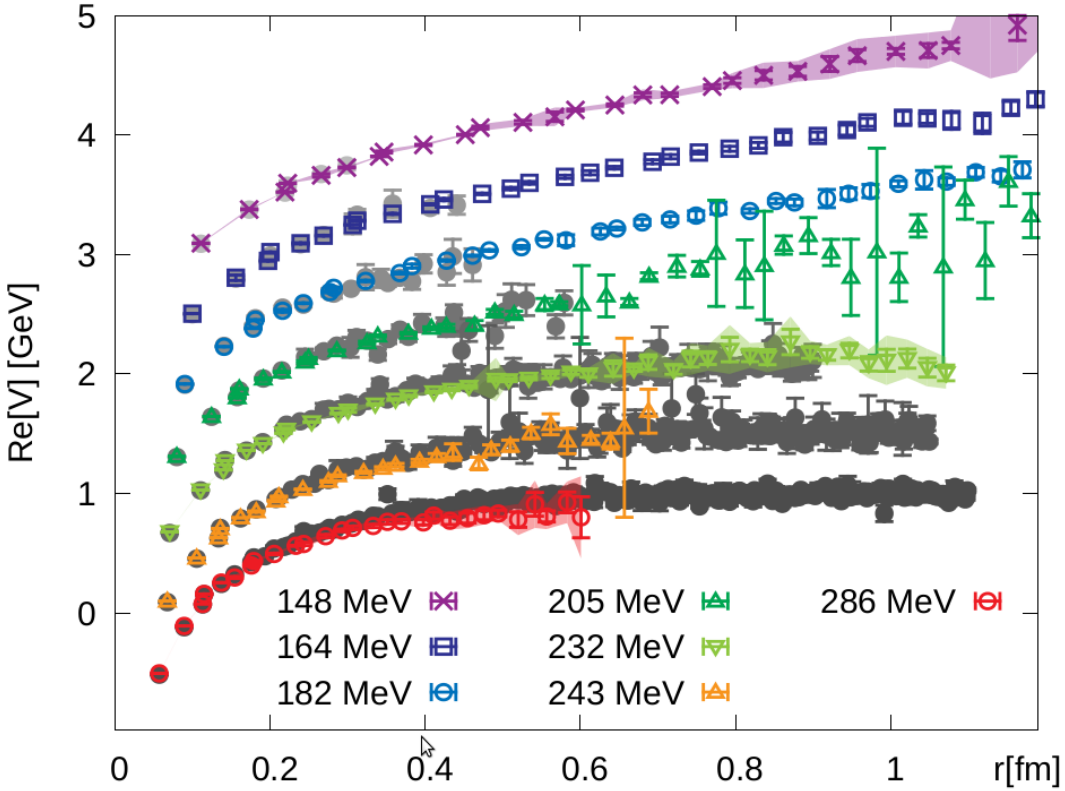

FIG. 3. Quark-gluon-plasma: The real part of the static interquark potential (open symbols) compared to the color singlet free energies in Coulomb gauge (gray circles).

Burnier, Kaczmarek, Rothkopf. **Static quark-antiquark potential in the quark-gluon plasma from lattice QCD** (2014)

Lattice QCD results

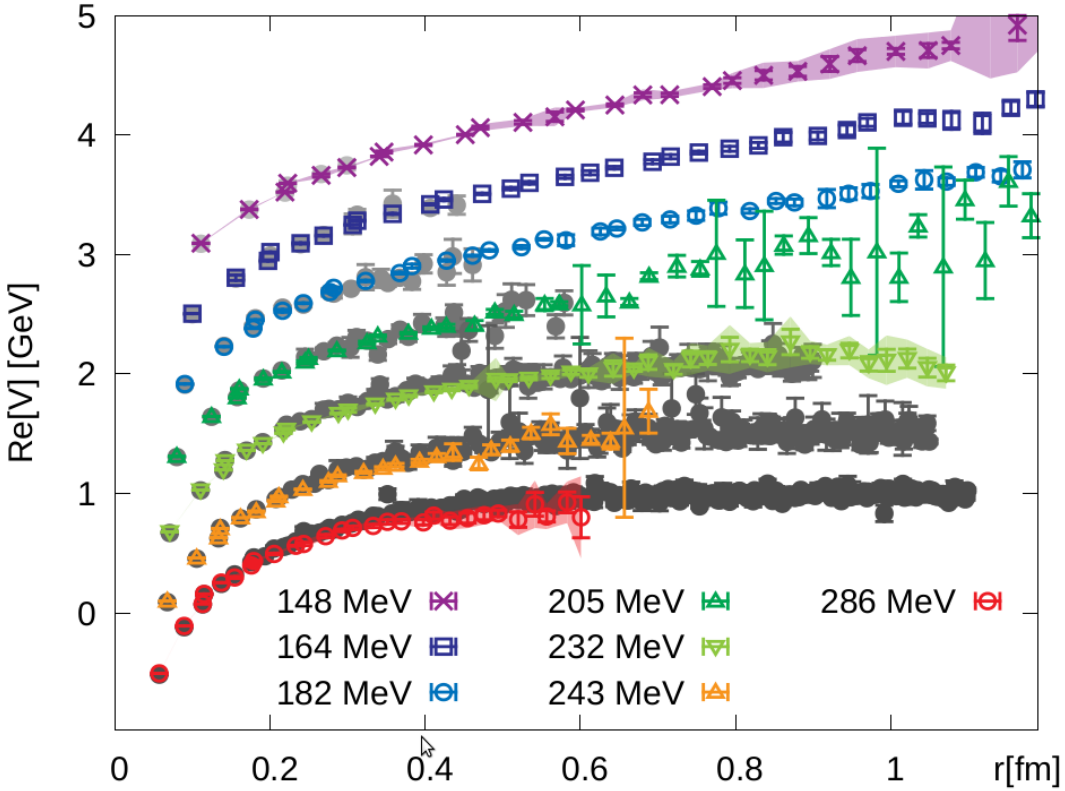

FIG. 3. Quark-gluon-plasma: The real part of the static interquark potential (open symbols) compared to the color singlet free energies in Coulomb gauge (gray circles).

$$\lambda_D(T) \sim 10^{-16}\ \mathrm{m}$$

Debye Radius QGP

$$\lambda_D \sim 10^{-16}\ \mathrm{m},$$

compare the diameter of a lead nucleus
(isotope 208 and 0.3 fm proton/neutron radius)

$$2 \cdot r_{\mathrm{Pb}} \sim \sqrt[3]{208} \cdot 0.3 \cdot 10^{-15} \approx 8 \cdot 10^{-15}\ \mathrm{m},$$

rounded favourably...

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Some tension?

$\Rightarrow$ edge effects

Debye number in QGP

?

Debye number in QGP (2)

Related to quark and gluon density in the medium. Also a big unknown.

Have found no claim on either N_D or quark/gluondensities. Of course linked to energy density for which there are estimates, but not sure how this translates into parton densities.

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Have found no claim on either N_D or quark/gluondensities. Of course linked to energy density for which there are estimates, but not sure how this translates into parton densities.

Of course the classical particle view does not work on this scale, but there should be an expectation value for the numberoperator within this sphere, right?

Section 3

Vlasov equations (of motion)

Conservation of particles

With particle density $f(\mathbf{r}, t)$. Conservation of number of particles states:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int f(\mathbf{r}, t)\mathrm{d}\mathbf{r} = \int_V \frac{\partial f(\mathbf{r}, t)}{\partial t}\mathrm{d}V + \int_S f(\mathbf{r}, t)\mathbf{v}\cdot\mathrm{d}\mathbf{S} = 0$$

Applying Gauss:

$$\int_V \left(\frac{\mathrm{d}\rho}{\mathrm{d}t} + \nabla\cdot(\rho\mathbf{v})\right) = 0$$

Phase space density function

The phase space density function $f(\mathbf{r}, \mathbf{v}, t)$ of particles is defined by

$$\mathrm{d}N = f(\mathbf{r}, \mathbf{v}, t)\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}.$$

Total number of particles in total volume is conserved, so

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\int f(\mathbf{r}, \mathbf{v}, t)\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v} \\ &= \int_{V6} \frac{\partial f}{\partial t}\mathrm{d}V + \int_{S3,\mathbf{r}} f(\mathbf{r}, \mathbf{v}, t)\mathbf{v}\cdot\mathrm{d}\mathbf{S}_{\mathbf{r}} + \int_{S3,\mathbf{v}} f(\mathbf{r}, \mathbf{v}, t)\mathbf{a}\cdot\mathrm{d}\mathbf{S}_{\mathbf{v}}\end{aligned}$$

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Applying Gauss:

$$\int_{V6}\left(\frac{\partial f}{\partial t} + \nabla\cdot(f\mathbf{v}) + \nabla_\mathbf{v}(f\mathbf{a})\right) = 0$$

Rewriting...

Since this is true for every 6-dimensional volume V,

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$$\mathbf{F} = q\mathbf{E} + \frac{\epsilon_0}{c}(\mathbf{v} \times \mathbf{B}), \text{ so } \nabla_{\mathbf{v}} \cdot \mathbf{F} = 0,$$

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so the $\mathbf{v}$ and $\mathbf{F}$ can be taken outside the brackets.
And if we split the forces on the plasma and in the plasma $F = F_{sv} + F_{coll}$, and rename

$$-\frac{\mathbf{F}_{\text{coll}}}{m} \cdot \nabla_{\mathbf{v}} f \equiv \left(\frac{\partial f}{\partial t}\right)_{\text{coll}},$$

Vlasov equation

we arrive at the Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e_0}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}},$$

with ∇ the space derivative and $\nabla_{\mathbf{v}}$ the momentum derivative.

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In case of rarified (diluted) plasma dominated by collective effects: collision term $\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} \approx 0$. This 'simplifies' to the Vlasov equation. Reasonable iff $\Lambda_{\text{Mean Free Path}} > r_D$.

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Bad news: Vlasov equation not analytically solvable (and numerically difficult as well).

Vlasov in QCD

In QCD:

- Three colors versus one e.m. charge
- Non-Abelian
- Quarks AND gluons
  (e.o.m. electrons decouple because of weight difference)

Vlasov in QCD (2)

$$
\begin{aligned}
[D_\nu, F^{\nu\mu}(x)]^a &= j^{\mu,a}_{\rm ind}, \\
j^{\mu,a}_{\rm ind} &= -\omega_D^2 \int \frac{d\Omega}{4\pi} v^\mu \Phi^a(x,\mathbf{v}), \\
[\mathbf{v}\cdot D_x, \Phi(x,\mathbf{v})]^a &= -\mathbf{v}\cdot\mathbf{E}^a(x), \\
\delta n^a_\pm(\mathbf{p},x) &= \pm g\Phi^a(x,\mathbf{v})\frac{\mathrm{d}n_{\rm F}}{\mathrm{d}p}, \\
\delta N^a(\mathbf{p},x) &= g\Phi^a(x,\mathbf{v})\frac{\mathrm{d}n_{\rm B}}{\mathrm{d}p}.
\end{aligned}
$$

Yagi, Hatsuda, Miake: **Quark Gluon Plasma: From Big Bang to Little Bang** (2005). or Blaizot, Iancu: **The Quark-Gluon Plasma: Collective Dynamics and Hard Thermal Loops** (2001)

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\delta N^a(\mathbf{p},x) &= g\Phi^a(x,\mathbf{v})\frac{{\rm d}n_{\rm B}}{{\rm d}p}.
\end{aligned}
$$

$n^a_\pm$ Distr. func. quarks. N^a Distr. func. gluons, and δ color oscilations
$D_\mu = \partial_\mu + igt^a A^a_\mu$ the convariant derivative.
Furthermore the $SU(3)$-generators t^a are hidden in $F_{\mu\nu} \equiv F^a_{\mu\nu} t^a$ and $\Phi = \Phi^a t^a$
n_F is the Fermi-Dirac, and n_B is the Boltzman distribution.

Section 4

Vlasov to fluid

Moments of Vlasov eqn.

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e_0}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}},$$

Multiply with m and integrating over $\mathbf{v}$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \cdot \mathbf{u}) = m\left(\frac{\partial n}{\partial t}\right)_{\text{coll}},$$

with $\mathbf{u}(\mathbf{r}, t)$ the bulk speed. If collisions do not alter the total mass, then $\left(\frac{\partial n}{\partial t}\right)_{\text{coll}} = 0$, and this is the mass continuity equation of fluid dynamics.

Moments of Vlasov eqn. (2)

Identically, multiplying with $m\mathbf{v}$ (and assuming momentum conservation) and $\frac{1}{2}m\mathbf{v}^2$ (energy conservation) yields:

$$\begin{aligned} \rho\frac{\mathrm{d}u_i}{\mathrm{d}t} &= \frac{\partial P_{ik}}{\partial x_k} + nF_i, \\ \frac{1}{\gamma-1}\rho^\gamma\frac{\mathrm{d}}{\mathrm{d}t}(P\rho^{-\gamma}) &= -\Pi_{ik}\frac{\partial u_i}{\partial x_k} - \frac{\partial q_k}{\partial x_k}, \end{aligned}$$

with pressure tensor P_{ik}, $P\delta_{ik}$ it's diagonal elements and Π_{ik} the off-diagonal elements. $\gamma = \frac{5}{3}$ for mono-atomic gas.

Hydro equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \cdot \mathbf{u}) &= 0, \\
\rho \frac{\mathrm{d} u_i}{\mathrm{d} t} &= \frac{\partial P_{ik}}{\partial x_k} + n F_i, \\
\frac{1}{\gamma - 1} \rho^{\gamma} \frac{\mathrm{d}}{\mathrm{d} t} (P \rho^{-\gamma}) &= -\Pi_{ik} \frac{\partial u_i}{\partial x_k} - \frac{\partial q_k}{\partial x_k}.
\end{aligned}
$$

Continuïty and Euler/Navier-Stokes equation.

Hydro equations

Non-relativistic fluid dynamics

The dynamical equations of motion

- Momentum conservation (or more generally, Newton's 2nd law) is expressed locally by...
  - the Euler equation

$$\rho(t,\vec{r})\Big[\partial_t\vec{\mathrm{v}}(t,\vec{r}) + \big[\vec{\mathrm{v}}(t,\vec{r})\cdot\vec{\nabla}\big]\vec{\mathrm{v}}(t,\vec{r})\Big] = -\vec{\nabla}\mathcal{P}(t,\vec{r}) + \vec{f}_\nu(t,\vec{r})$$

☞ if the fluid is "perfect" (or "ideal")

  - the Navier-Stokes equation

$$\rho(t,\vec{r})\Big[\partial_t\vec{\mathrm{v}}(t,\vec{r}) + \big[\vec{\mathrm{v}}(t,\vec{r})\cdot\vec{\nabla}\big]\vec{\mathrm{v}}(t,\vec{r})\Big] = -\vec{\nabla}\mathcal{P}(t,\vec{r}) + \vec{f}_{\mathrm{visc.}}(t,\vec{r}) + \vec{f}_\nu(t,\vec{r})$$

☞ if the fluid is "Newtonian"

  - the Burnett / super-Burnett equation... ... if the fluid is...

not kidding!

Topical Lectures, NIKHEF, June 15-17, 2015 N.Borghini — I-16/58 Universität Bielefeld

See Topical Lectures by Nicolas Borghini.

Section 5

Plasma frequency

Plasma Frequency (E.M.)

Small pertubation of electrons in a plasma:

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} = -\frac{e}{m_e}E = -\frac{e^2\overline{n}}{\epsilon_0 m_e}\xi.$$

Harmonic oscilation:

$$\omega_p = \sqrt{\frac{\overline{n}e^2}{\epsilon_0 m_e}}$$

known as **plasma frequency**. With $v_e = \sqrt{\frac{kT_e}{m_e}}$ the electron thermal speed. $r_D\omega_p = v_e$.

Plasma Frequency (E.M.)

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Typical **thermal** scales in a plasma:

- Length: Debye length
- Particles: Debye number
- Time: One over the plasma frequency
- Speed: Electron thermal speed

This table again

Plasma	n_e (m^{-3})	T (K)	B (T)	λ_D (m)	N_D	ω_p (s^{-1})
Gas discharge	10^{16}	10^4	—	10^{-4}	10^4	10^{10}
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`http://www.cns.gatech.edu/PHYS-4421/caltech136/` (2000)

Plasma Frequency (QGP)

Can be calculated in perturbative QCD. In a gluon plasma it turned out to be

$$\omega_p^2 = m^2(1 + \eta g\sqrt{N}),$$

in next-to-leading order, where $m^2 = \frac{1}{9}g^2NT^2$, with N the number of colors, T the temperature, g the strong coupling and $\eta = -0.18$.

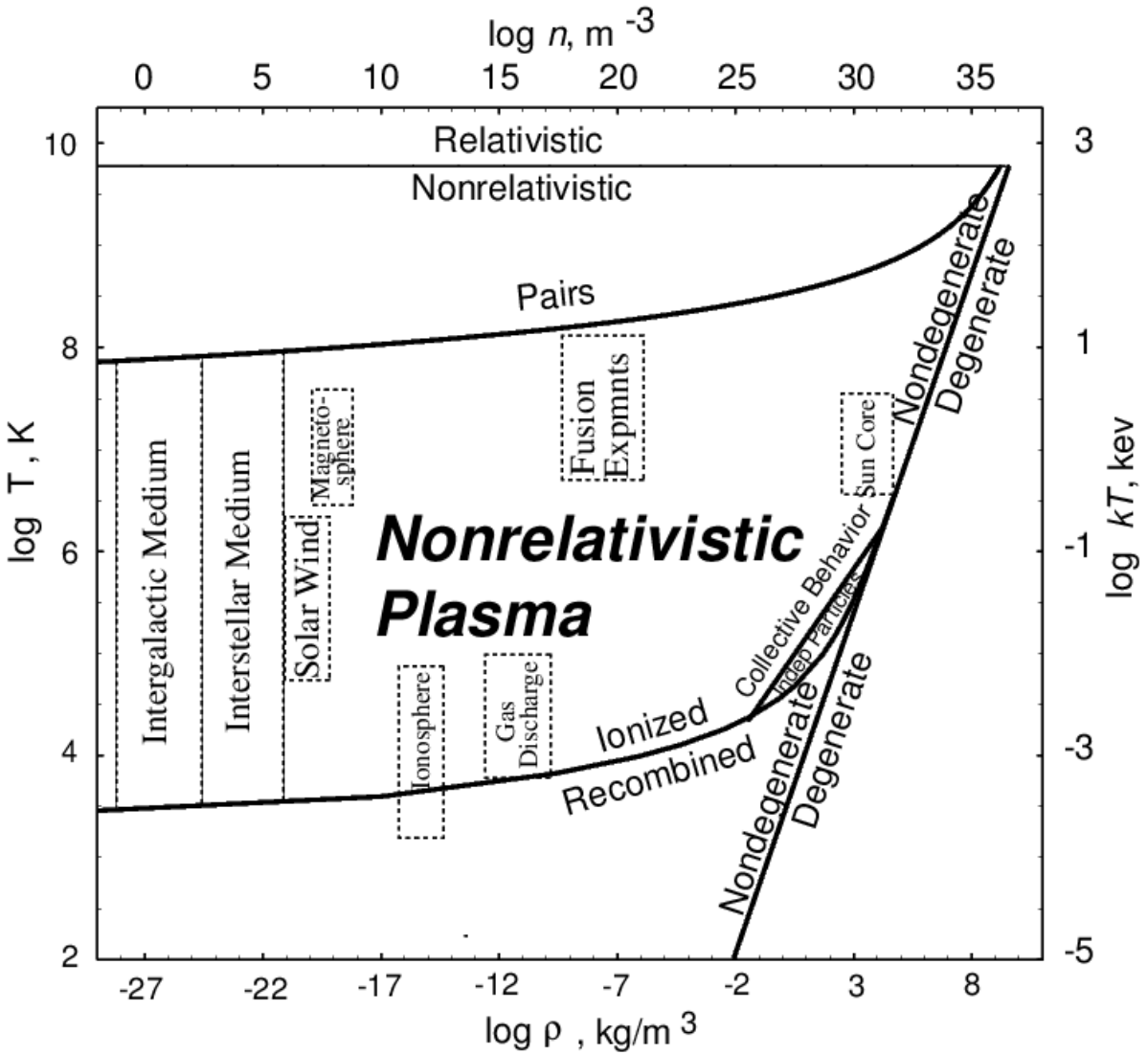
Schulz, Nuclear Physics B413 (1994)

But slow convergence, so only valid for **high** temperature.

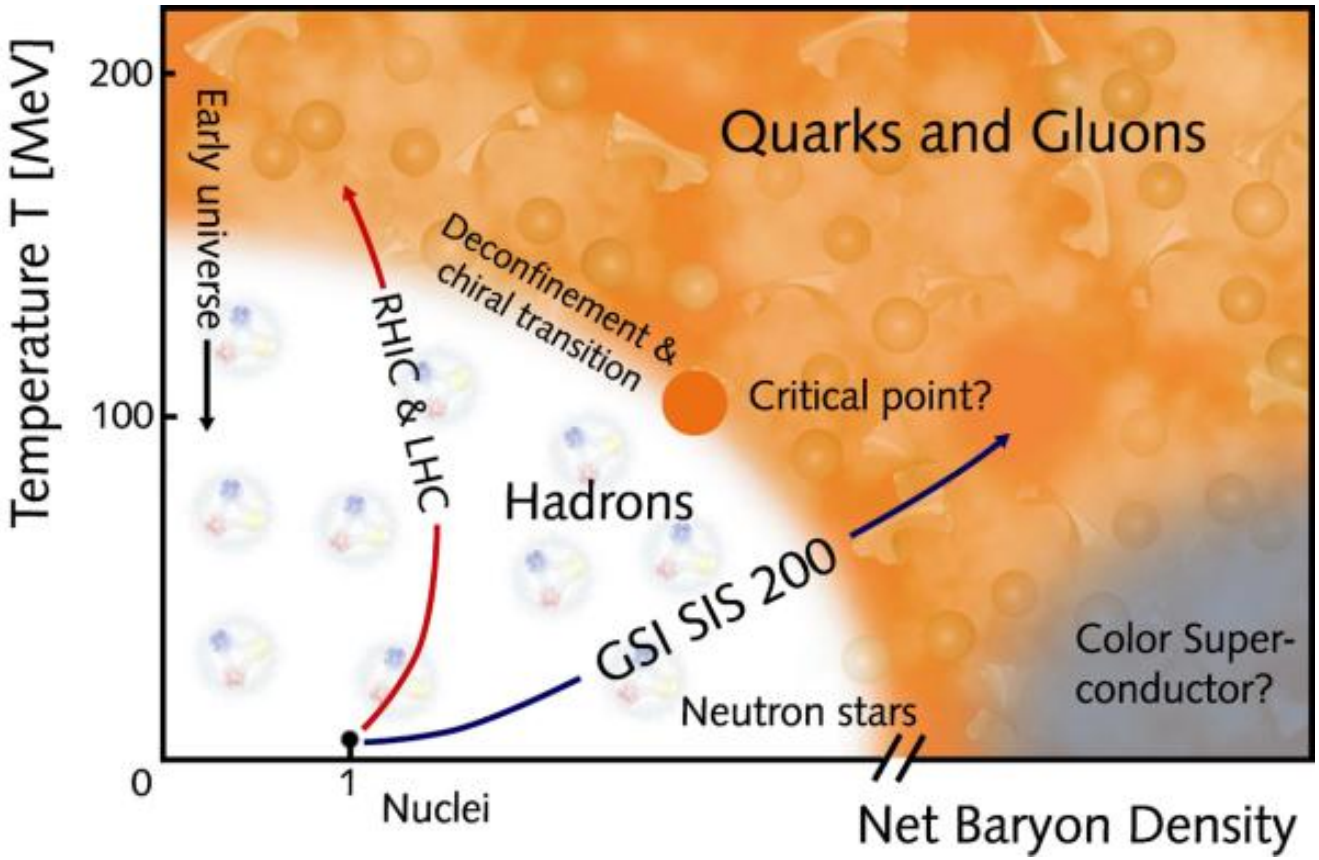
Section 6

Phase diagram

Phase Diagram EMP



Phase Diagram QGP



Section 7

Summary

Summary

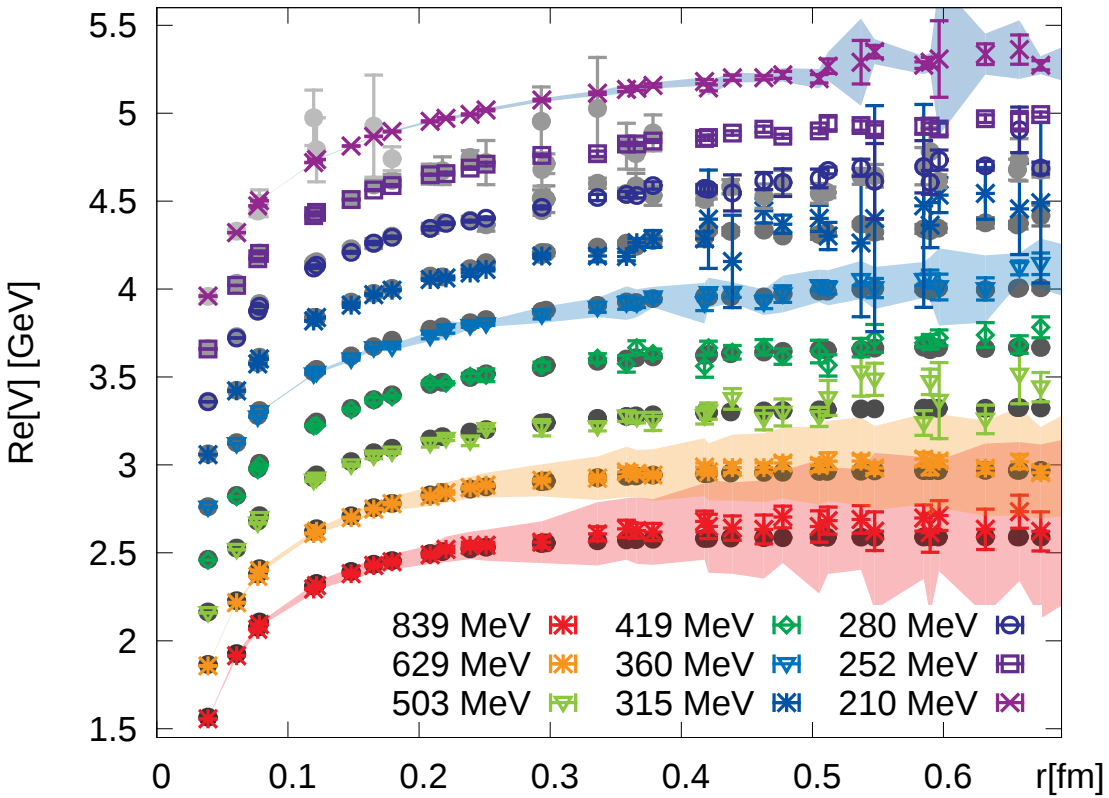
- Looked at screening within QED and QCD
- Looked at the typical quantities of a plasma in QED and QCD
  - Debye Length
  - Debye Number
  - Plasma Frequency
  - Thermal Speed
- Derived Vlasov equations for E.M. plasma
- Witnessed that QGP makes it more difficult
- Explained how this leads to the hydronamics equations
- Phase Diagrams

The end

Questions?

P.S. If you are looking for a topic for the Journal club yourself. Waves in plasma might be a nice and not too broad idea like this...

Lattice QCD results



"Gluonic medium: The shifted real part of the static inter-quark potential (open symbols) compared to the color singlet free energies (gray circles)."

Burnier, Kaczmarek, Rothkopf. **Static quark-antiquark potential in the quark-gluon plasma from lattice QCD** (2014)